Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:________________________________

1. Find the point(s) on the curve $y^3 = x^2$ closest to the point $(0, 4)$.

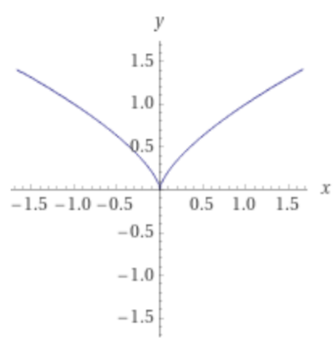
To receive full credit for this problem, you must show all 7 steps, as discussed in Lesson 24-26.
Hints:

(a) Step 2: Copy the graph from wolfram alpha
(b) Step 5: Determine the domain with the graph in Step 2.
(c) Step 6: When determining the absolute extrema, remember to check your endpoint(s) too.

Solution: **Steps:**

(1) Distance

(2) Graph



(3) $D = (x-0)^2 + (y-4)^2 = x^2 + (y-4)^2$

(4) $y^3 = x^2$

(5) Domain of x: $(-\infty, \infty)$
Domain of y: $[0, \infty)$

(6) Plug (4) into (3).

$$D = x^2 + (y-4)^2$$
$$= y^3 + y^2 - 8y + 16$$

Take the derivative and set = 0.

$$D' = 3y^2 + 2y - 8 = 0$$
$$3y^2 + 6y - 4y - 8 = 0$$
$$3y(y+2) - 4(y+2) = 0$$
$$(3y-4)(y+2) = 0$$
$$y = 4/3, -2$$

Now we need to check for absolute minimum, via the Second Derivative Test. So,

$$D'' = 6y + 2$$

Note that $y = -2$ is not in domain of y. So we will use $x = 0$ (because it is an endpoint) and $x = 4/3$ for the Test.

$$D''(0) = 6(0) + 2 = 2$$
$$D''(4/3) = 6(4/3) + 2 = 10$$

Since 2 is the smallest of the two values, the absolute min happens there. i.e. The closest point on the graph to (0,4) occurs at $y = 0$.

(7) Recall what the problem is asking you "the point on the curve closest to the point (0,4)." If $y = 0$, then

$$0^3 = x^2$$
$$x = 0$$

So the point is $(0, 0)$.