Formulas

Product Rule: $f(x) = u(x)v(x)$

$$f'(x) = u'(x)v(x) + v'(x)u(x)$$

Quotient Rule: $f(x) = \dfrac{u(x)}{v(x)}$

$$f'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{v^2(x)}$$

Power Rule: $f(x) = x^n$ where n is a #

$$f'(x) = nx^{n-1}$$

Tangent Line:
1) Find y'
2) Plug $x=c$ for y, and y'.
3) Plug values from 2 into
$$y - y(c) = y'(c)(x-c)$$
4) Solve for y.

Derivatives of Trig Functions

$\frac{d}{dx}[\sin x] = \cos x$ $\qquad$ $\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$ $\qquad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\qquad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Continuity: $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c) = \lim_{x \to c} f(x)$

Limit doesn't exist (DNE) when $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

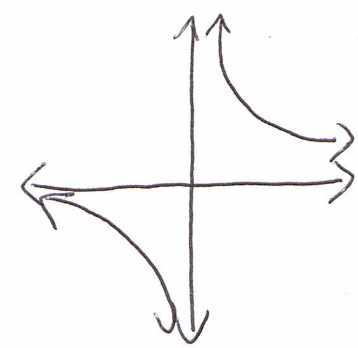
Ex: $f(x) = \frac{1}{x}$

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

$\lim_{x \to 0^+} \frac{1}{x} = \infty$

$\lim_{x \to 0} \frac{1}{x}$ DNE

$f(0)$ undefined



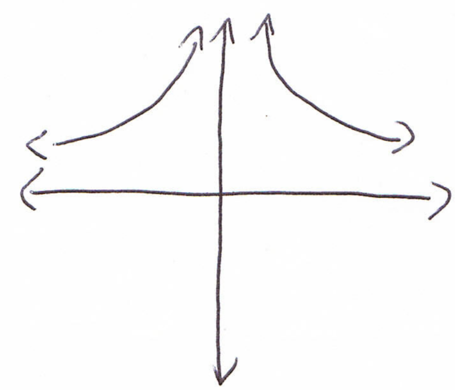
Ex. $f(x) = \frac{1}{x^2}$

$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$

$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$

$\lim_{x \to 0} \frac{1}{x^2} = \infty$

$f(0)$ undefined



Rate of change is the derivative

~~Def~~ Limit Def of Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Horizontal Tangent means the derivati[ve] is zero.

$f'(x) = 0$

E1P2

- $(a \pm b)^2 = a^2 \pm 2ab + b^2$
- $x^{-m} = \frac{1}{x^m}$
- $a^2 - b^2 = (a+b)(a-b)$
- $\sqrt[q]{x^p} = x^{p/q}$

Trick to multiply terms

$$h(x) = (2x+1)(3x^2+2x+1) = 6x^3 + 7x^2 + 4x + 1$$

	$3x^2$	$2x$	1
$2x$	$6x^3$	$4x^2$	$2x$
1	$3x^2$	$2x$	1

$\tan x = \frac{\sin x}{\cos x}$ $\csc x = \frac{1}{\sin x}$

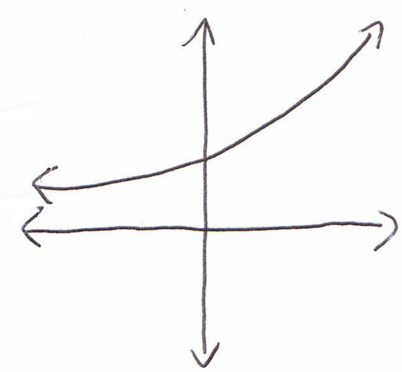
$\cot x = \frac{\cos x}{\sin x}$ $\sec x = \frac{1}{\cos x}$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	$0 = \frac{\sqrt{0}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
cos	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{0}{2} = 0$

Remember

$\boxed{e^x > 0}$

Never! $e^x \neq 0$



Vertical Asymptote vs. Hole

Hole when factors cancel out. $f(x) = \frac{x^2(x-4)}{x-4}$

If no cancellation, then VA. $f(x) = \frac{1}{x}$

E1P3

Recipe for Solving a Related Rates Problem

Step 1: Draw a good picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you **WANT** to find.

Step 3: Find an equation relating the relevant variables. This usually involves a formula from geometry, similar triangles, the Pythagorean Theorem, or a formula from trigonometry. Use your picture!

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t*.

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you **WANT**. Do **NOT** substitute before this step!

Some Useful Formulas

Right Triangle
Pythagorean Theorem:
$a^2 + b^2 = c^2$

Triangle
$A = \frac{1}{2}bh$
$P = a + b + c$

Equilateral Triangle
$h = \frac{\sqrt{3}}{2}s$ $A = \frac{\sqrt{3}}{4}s^2$

Rectangle
$A = lw$
$P = 2l + 2w$

Trapezoid
$A = \frac{1}{2}(a+b)h$

Circle
$A = \pi r^2$ $C = 2\pi r$

Circular Sector
$A = \frac{1}{2}r^2\theta$ $s = r\theta$

Circular Ring
$A = \pi(R^2 - r^2)$

Rectangular Box
$V = lwh$
$S = 2(hl + lw + hw)$

Sphere
$V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

Right Circular Cylinder
$V = \pi r^2 h$ $S = 2\pi rh$

Right Circular Cone
$V = \frac{1}{3}\pi r^2 h$

E2P1

Position/Velocity/Acceleration

Position	$s(t)$
Velocity	$v(t) = s'(t)$
Acceleration	$a(t) = v'(t) = s''(t)$

Chain Rule:

$f(x) = a(b(x))$

$f'(x) = a'(b(x))\, b'(x)$

Derivatives of Exponential & Logarithmic Functions

$\frac{d}{dx}[e^x] = e^x$ $\qquad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}$

Higher Order Derivatives

Idea: Take the derivative of the derivative.

Implicit Differentiation

Idea: Take the derivative of each side. Using $\frac{d}{dx}$.
Solve for $\frac{dy}{dx}$.

Useful Formulas

$\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$ $\qquad$ $\frac{d}{dx}\left(\frac{x}{y}\right) = \frac{y - x\frac{dy}{dx}}{y^2} = \frac{1}{y} - \frac{x}{y^2}\frac{dy}{dx}$

Critical Numbers:

1. Find the domain of y
2. Find out when $f'(x) = 0$ or $f'(x)$ is undefined.
3. Check that the x-values found in 2 don't match 1.
   If they do, then that value isn't a critical number.

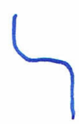
Increasing if $f'(x) > 0$ on an interval
Decreasing if $f'(x) < 0$ on an interval

The First Derivative Test: c is a critical #.

① $+ \; \underset{c}{|} \; -$ f' ex. $\cap$ $\Rightarrow$ rel max @ $x = c$

② $- \; \underset{c}{|} \; +$ f' ex. $\cup$ $\Rightarrow$ rel min @ $x = c$

③ $+ \; \underset{c}{|} \; +$ f' ex.  $\Rightarrow$ neither

④ $- \; \underset{c}{|} \; -$ f' ex.  $\Rightarrow$ neither

How to Determine Increasing/Decreasing & Relative Extrema

① Find critical pts.
② Draw a # line with the points from ①.
③ Determine test pts using ②.
④ Plug those values into f' to determine whether its positive or negative.
We don't actually care about the value. Only whether it is + or −.
⑤ Use definition of increasing/decreasing and the First Derivative Test to determine
ⓐ Increasing/Decreasing
ⓑ Relative Extrema

Remember $\sin^2 x = [\sin x]^2 \neq \sin(x^2)$

Logarithmic Properties

ⓐ $\ln e^x = x$

ⓑ $\ln(ab) = \ln(a) + \ln(b)$

ⓒ $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

ⓓ $\ln(a^x) = x\ln(a)$

Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a-c method for factoring

ex. $\underbrace{4}_{a}x^2 \underbrace{-4}_{b}x \underbrace{-3}_{c}$

First find $ac = 4 \cdot (-3) = -12$

Next list all factors of $ac = -12$

1 12
2 6
3 4

The idea is to add the factors to yield $b = -4$

Note since $ac = -12$ and $b = -4$ the largest factor gets a negative. So

$ac = -12$

$1 - 12 = -11$

$\boxed{2 - 6 = -4 = b}$

$3 - 4 = -1$

E2P4

Rewrite the middle term with the numbers in the pink box.

$$4x^2 - 4x - 3 = 4x^2 + 2x - 6x - 3$$

Now factor by grouping.

$$= 2x(2x+1) - 3(2x+1)$$

Check that the parenthesis match.

$$= (2x-3)(2x+1)$$