| Differentiation Rules | Integration Rules |
|---|---|
| $\frac{d}{dx}(c) = 0$ | $\int 0\,dx = C$ |
| $\frac{d}{dx}(kx) = k$ | $\int k\,dx = kx + C$ |
| $\frac{d}{dx}(kf(x)) = kf'(x)$ | $\int kf'(x)\,dx = kf(x) + C$ |
| $\frac{d}{dx}(x^n) = nx^{n-1}$ | $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ when $n \neq -1$ |
| $\frac{d}{dx}(\sin x) = \cos x$ | $\int \cos x\,dx = \sin x + C$ |
| $\frac{d}{dx}(\cos x) = -\sin x$ | $\int \sin x\,dx = -\cos x + C$ |
| $\frac{d}{dx}(\tan x) = \sec^2 x$ | $\int \sec^2 x\,dx = \tan x + C$ |
| $\frac{d}{dx}(\cot x) = -\csc^2 x$ | $\int \csc^2 x\,dx = -\cot x + C$ |
| $\frac{d}{dx}(\sec x) = \sec x \tan x$ | $\int \sec x \tan x\,dx = \sec x + C$ |
| $\frac{d}{dx}(\csc x) = -\csc x \cot x$ | $\int \csc x \cot x\,dx = -\csc x + C$ |
| $\frac{d}{dx}(e^x) = e^x$ | $\int e^x\,dx = e^x + C$ |
| $\frac{d}{dx}(\ln x) = \frac{1}{x},\ x > 0$ | $\int \frac{1}{x}\,dx = \ln|x| + C$ |

- For Exam 4, we can ignore the +C for the integration rules because we have upper and lower bound.

$$\int_a^b f(x)\,dx = \text{Signed Area}$$

- Always take derivative of your answer, before evaluating when integrating. Especially when you have trig functions!!!

E4P1

## Signed Area: Area enclosed by $f(x)$ and x-axis.

- If $f(x)$ <u>above</u> the x-axis, then area is <u>positive</u>.
- If $f(x)$ <u>below</u> the x-axis, then area is <u>negative</u>.

## Riemann Sums

<u>Left</u>: $L_n = \sum\limits_{i=0}^{n-1} f(x_i) \Delta x$      <u>Right</u>: $R_n = \sum\limits_{i=1}^{n} f(x_i) \Delta x$

where $x_i = a + i\Delta x$ and $\Delta x = \dfrac{b-a}{n}$

Tips: ① Determine what $a$, $b$, and $n$ are.
② Calculate the following in the following order:
    ⓘ $\Delta x$    �ⅱ $x_i$    ⅲ $f(x_i)$    ⅳ $L_n$ or $R_n$

## Trapezoid Rule

$$T_n = \frac{1}{2}\Delta x\left(f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)\right)$$

where $x_i = a + i\Delta x$ and $\Delta x = \dfrac{b-a}{n}$

Tips: ① Determine what $a$, $b$, and $n$ are.
② Calculate the following in the following order:

| ⓘ $\Delta x$ | ⅱ $x_0$ | ⅲ $f(x_0)$ | ⅳ $f(x_0)$ |
|---|---|---|---|
| | $x_1$ | $f(x_1)$ | $2f(x_1)$ |
| | $\vdots$ | $\vdots$ | $\vdots$ |
| | $x_{n-1}$ | $f(x_{n-1})$ | $2f(x_{n-1})$ |
| | $x_n$ | $f(x_n)$ | $f(x_n)$ |

③ Sum all values from ⅳ and multiply by $\frac{1}{2}\Delta x$ Yielding $T_n$.

When given the graph of a shaded region, to determine the integral,

① Left-most x-value $\Rightarrow a$

② Right-most x-value $\Rightarrow b$

$$\int_a^b f(x)\,dx$$

③ the line connecting them $\Rightarrow f(x)$

↳ Use algebra to determine the equation

## Properties of Definite Integrals

① $\int_a^a f(x)\,dx = 0$   ② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

④ $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

⑤ $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

## FTC: $\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$

## Exponential Growth/Decay Models

$$\frac{dy}{dt} = y' = ky \Rightarrow y = Ce^{kt}$$   where   $k$ - growth rate $\Big\}$ Constant
                                                          $C$ - initial value

Growth $\Rightarrow k > 0$    Decay $\Rightarrow k < 0$

Half-Life $\Rightarrow k = -\dfrac{\ln(2)}{\text{half-life}}$   or   $k = \dfrac{\ln(\frac{1}{2})}{\text{half-life}}$

E4P3

- $(a \pm b)^2 = a^2 \pm 2ab + b^2$

- $a^2 - b^2 = (a-b)(a+b)$

## Exponential Rules (Note these rules apply when $x = e$)

- $x^{a+b} = x^a x^b$

- $x^{-m} = \dfrac{1}{x^m}$

- $x^0 = 1$

- $x^{a-b} = \dfrac{x^a}{x^b}$

- $x^{p/q} = \sqrt[q]{x^p}$

## Logarithmic Properties

- $\ln e^x = x = e^{\ln x}$

- $\ln(ab) = \ln(a) + \ln(b)$

- $\ln(a^x) = x \ln a$

- $\ln\left(\dfrac{a}{b}\right) = \ln(a) - \ln(b)$

## Trig Identities

$\tan x = \dfrac{\sin x}{\cos x}$     $\cot x = \dfrac{\cos x}{\sin x}$     $\csc x = \dfrac{1}{\sin x}$     $\sec x = \dfrac{1}{\cos x}$

|       | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|-------|---|---------|---------|---------|---------|
| Sin   | $0 = 0/2$ | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | $\sqrt{4}/2 = 1$ |
| Cos   | $1 = \sqrt{4}/2$ | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | $0/2 = 0$ |

Remember $\sin^2 x = [\sin x]^2 \neq \sin(x^2)$

## Trick to multiplying terms

$h(x) = (2x+1)(3x^2 + 2x + 1) = 6x^3 + 4x^2 + 3x^2 + 2x + 2x + 1$
$$= 6x^3 + 7x^2 + 4x + 1$$

|      | $3x^2$ | $2x$ | $1$ |
|------|--------|------|-----|
| $2x$ | $6x^3$ | $4x^2$ | $2x$ |
| $1$  | $3x^2$ | $2x$ | $1$ |