Lesson 10: Quotient Rule; Derivatives of Other Trigonometric Functions

Recall Product Rule

$$h(x) = u(x)v(x)$$

$$h'(x) = u'(x)v(x) + v'(x)u(x)$$

<u>Quotient Rule</u>

$$h(x) = \frac{u(x)}{v(x)}$$

$$h'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{v^2(x)}$$

<u>Ex 1:</u> $h(x) = \frac{1}{x^2}$. Find $h'(x)$.

<u>Method 1:</u> Power Rule

$$h(x) = x^{-2}$$

$$h'(x) = -2x^{-3} = -\frac{2}{x^3}$$

<u>Method 2:</u> Quotient Rule

$u(x) = 1$ $\quad$ $v(x) = x^2$

$u'(x) = 0$ $\quad$ $v'(x) = 2x$

$$h'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{v^2(x)}$$

$$= \frac{\cancel{0 \cdot x^2}^{\,0} - 2x \cdot 1}{(x^2)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

<u>Ex 2:</u> $y = \dfrac{x^2+1}{x^3-3x}$. Find y'.

$u(x) = x^2+1 \qquad v(x) = x^3 - 3x$

$u'(x) = 2x \qquad v'(x) = 3x^2 - 3$

$$y' = \frac{2x(x^3-3x) - (3x^2-3)(x^2+1)}{(x^3-3x)^2}$$

$2x(x^3-3x) = 2x^4 - 6x^2$

$(x^2+1)(3x^2-3) = 3x^4 - 3$

	x^2	1
$3x^2$	$3x^4$	$\cancel{3x^2}$
-3	$\cancel{-3x^2}$	-3

$(x^3-3x)^2 = (x^3)^2 - 2(x^3)(3x) + (3x)^2$
$= x^6 - 6x^4 + 9x^2$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$$h'(x) = \frac{2x^4 - 6x^2 - (3x^4 - 3)}{x^6 - 6x^4 + 9x^2}$$

$$= \frac{-x^4 - 6x^2 + 3}{x^6 - 6x^4 + 9x^2}$$

Ex 3: $f(x) = \dfrac{\sin x}{x + \sin x}$. Find f'.

$u(x) = \sin x \qquad v(x) = x + \sin x$

$u'(x) = \cos x \qquad v'(x) = 1 + \cos x$

$$f'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{v^2(x)}$$

$$= \frac{\cos x(x + \sin x) - (1 + \cos x)\sin x}{(x + \sin x)^2}$$

$$= \frac{x\cos x + \cancel{\cos x \sin x} - \sin x - \cancel{\sin x \cos x}}{(x + \sin x)^2}$$

$$= \frac{x\cos x - \sin x}{(x + \sin x)^2}$$

Derivatives of other Trig Functions

$\frac{d}{dx}[\sin x] = \cos x$ $\qquad$ $\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\tan x] = \sec^2 x$ $\qquad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\qquad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

To prove these use quotient Rule.
For example,

$$\tan x = \frac{\sin x}{\cos x} \longrightarrow \frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

HW 10.11: $y = 3\sin x \tan x$

$u(x) = 3\sin x \qquad v(x) = \tan x$

$u'(x) = 3\cos x \qquad v'(x) = \sec^2 x$

$$y' = u'(x)v(x) + v'(x)u(x)$$

$$= 3\cos x \tan x + \sec^2 x \cdot 3\sin x$$

$$= 3\cancel{\cos x}\,\frac{\sin x}{\cancel{\cos x}} + \frac{1}{\cos^2 x}\cdot 3\sin x$$

$$= 3\sin x + 3\sec^2 x \sin x$$

<u>HW 10.6</u>: $f(t) = \dfrac{e^t}{4-e^t}$. Find $f'(8)$.

$u(t) = e^t \qquad v(t) = 4 - e^t$

$u'(t) = e^t \qquad v'(t) = -e^t$

$$f'(t) = \frac{u'(t)\,v(t) - v'(t)\,u(t)}{v^2(t)}$$

$$= e^t(4-e^t) + (+e^t)\,e^t$$