Lesson 13: Higher Order Derivatives

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The derivative of a function, $f(x)$, is also called the first derivative.

$$y', \; f'(x), \; \frac{dy}{dx}, \; \frac{d}{dx}[f(x)]$$

If we take the derivative of the first derivative of $y=f(x)$, then we get second derivative.

$$y'', \; f''(x), \; \frac{d^2y}{dx^2}, \; \frac{d^2}{dx^2}[f(x)]$$

Take the derivative n times, I get the nth derivative.

$$y^{(n)}, \; f^{(n)}(x), \; \frac{d^ny}{dx^n}, \; \frac{d^n}{dx^n}[f(x)]$$

Ex 1: Given $f(x) = xe^x$. Find $f''(x)$ and $f^{(3)}(x)$.

First find f'.

$u(x) = x \qquad v(x) = e^x$
$u'(x) = 1 \qquad v'(x) = e^x$

$$f'(x) = u'v + v'u$$
$$= 1 \cdot e^x + e^x \cdot x$$
$$= e^x(1+x)$$

Find f''. (Taking the derivative of f').

$u(x) = e^x \qquad v(x) = 1+x$
$u'(x) = e^x \qquad v'(x) = 1$

$$f''(x) = u'v + v'u$$
$$= e^x(1+x) + 1 \cdot e^x$$
$$= e^x(1+x+1)$$
$$= e^x(2+x)$$

Find $f^{(3)}(x)$. (Taking the derivative of f'').

$u(x) = e^x \qquad v(x) = 2+x$
$u'(x) = e^x \qquad v'(x) = 1$

$$f^{(3)}(x) = u'v + v'u$$
$$= e^x(2+x) + 1 \cdot e^x$$

$$= e^x(2+x+1)$$
$$= e^x(3+x)$$

Position/Velocity/Acceleration Functions

position $s(t), h(t)$

velocity $v(t)$

$$\boxed{v(t) = s'(t)}$$

acceleration $a(t)$

$$\boxed{a(t) = v'(t) = s''(t)}$$

Ex 2: Position is

$$s(t) = \frac{1}{12}t^4 - \frac{4}{3}t^3 + 8t^2 - 64t$$

Find $a(2)$. $[s''(2)]$

$$s'(t) = \frac{4}{12}t^3 - \frac{4}{3}\cdot 3t^2 + 8\cdot 2t - 64$$
$$= \frac{1}{3}t^3 - 4t^2 + 16t - 64$$