Lesson 17: Relative Extrema and Critical Numbers

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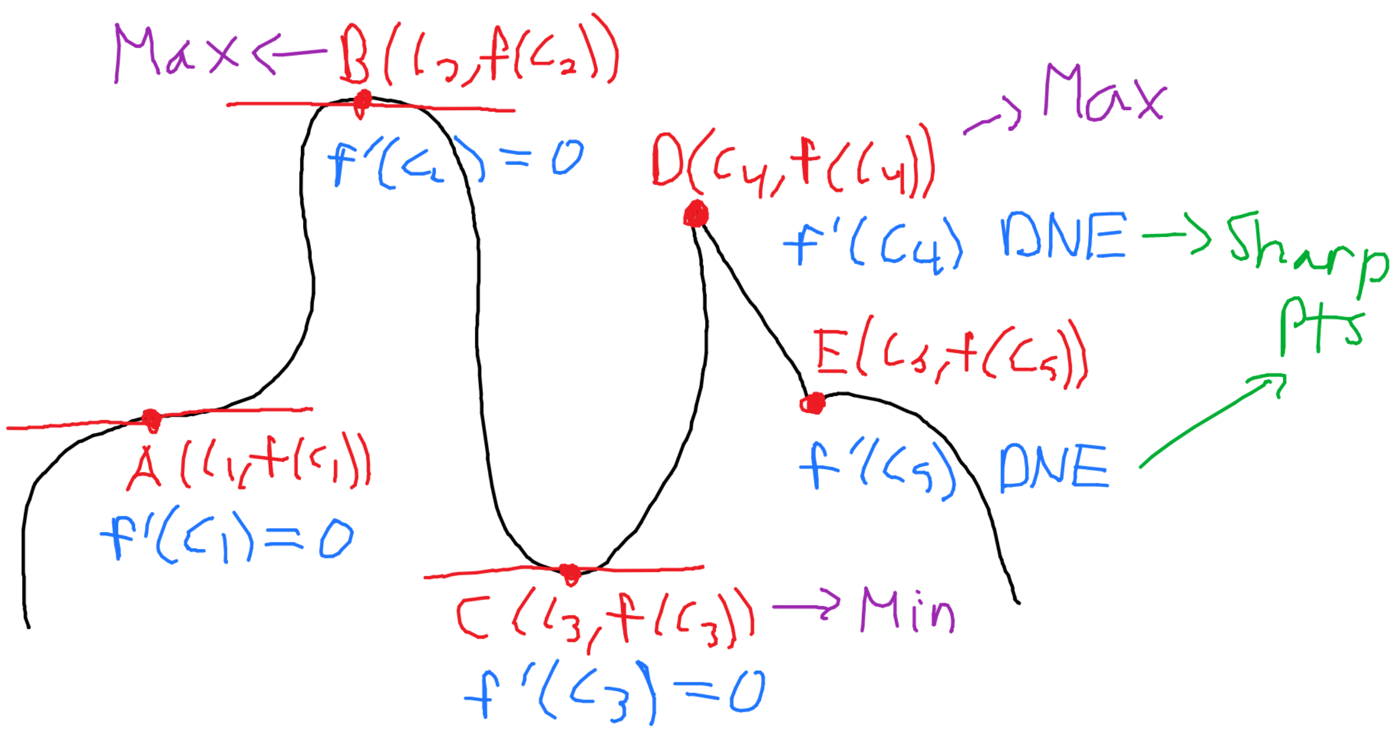
Def: (a) If $\boxed{f(c) \geq f(x)}$ for all x in open interval I containing c, then $f(c)$ is a $\boxed{\text{relative maximum}}$.

(b) If $\boxed{f(c) \leq f(x)}$ for all x in open interval I containing c, then $f(c)$ is a $\boxed{\text{relative minimum}}$.



Overall, all relative extrema occur at points where the derivative is zero or DNE.

Question: If the derivative is zero or DNE, do we have a relative extrema?

Answer: No b/c pts A and E, from the graph

Def: Let c be a # in the domain of f. If $f'(x)=0$ or $f'(x)$ DNE @ $x=c$ then c is a critical number.

Ex 1: Find the critical #s of

$$y = x^4 - 2x^3$$

Idea: Solve $y'(x)=0$ for x.

$$y' = 4x^3 - 6x^2 = 0$$

$$2x^2(2x-3)=0$$

$x=0$ $\quad$ $x=\frac{3}{2}$ $\longrightarrow$ critical #s.

Ex 2: Find the critical #s of

$$y = 5x^{4/5}$$

$y' = 5 \cdot \frac{4}{5} x^{-1/5} = 0$

$4x^{-1/5} = 0$

$\frac{4}{x^{1/5}} = \frac{0}{1}$

$4 = 0x^{1/5}$

$4 = 0 \rightarrow y'(x) \neq 0$

When does y' DNE?

$y' = \frac{4}{x^{1/5}}$

$x = 0$ when y' DNE

$\downarrow$

Critical pt

Ex 3: Find the critical #s of

$$y = 3x^4 e^x$$

$\boxed{e^x \neq 0 \ \forall x}$

for all

$u(x) = 3x^4 \quad v(x) = e^x$

$u'(x) = 12x^3 \quad v'(x) = e^x$

$y' = 12x^3 e^x + e^x(3x^4) = 0$

$\underbrace{3x^3}_{x=0} \underbrace{e^x}_{\text{Nothing}} \underbrace{(4+x)}_{x=-4} = 0$

Critical #s are
$x = 0, x = -4$

Ex 4: Find the critical #s of

$$y = 2x^3 e^{2x+1}$$

$\boxed{e^x \neq 0}$

$u(x) = 2x^3 \qquad v(x) = e^{2x+1}$

$u'(x) = 6x^2 \qquad v'(x) = e^{2x+1}(2) = 2e^{2x+1}$

$$y' = 6x^2 e^{2x+1} + 2x^3(2e^{2x+1}) = 0$$

$$\underbrace{2x^2}_{x=0}\ \underbrace{e^{2x+1}}_{\text{Nothing}}\ \underbrace{[3+2x]}_{x=-\frac{3}{2}} = 0$$

Critical Numbers
$x = 0, -3/2$

Ex 5: Find the critical #s of

$$y = \frac{4x^2+3}{2x+1}$$

First find domain of y: $2x+1 \neq 0$
$x = -1/2$

Next, find y': $u(x) = 4x^2+3 \qquad v(x) = 2x+1$
$u'(x) = 8x \qquad v'(x) = 2$

$$y' = \frac{8x(2x+1) - 2(4x^2+3)}{(2x+1)^2} = 0$$

$$\frac{16x^2 + 8x - 8x^2 - 6}{(2x+1)^2} = 0$$

$$\frac{8x^2+8x-6}{(2x+1)^2}=0$$

$$8x^2+8x-6=0$$
$$2(4x^2+4x-3)=0$$
$$4x^2+4x-3=0$$

Check for y' DNE

$$(2x+1)^2=0$$
$$x=-1/2$$

↳ Not a critical # b/c $-1/2$ isn't in domain of y.

$$4x^2+4x-3=0$$
$$4x^2-2x+6x-3=0$$
$$2x(2x-1)+3(2x-1)=0$$
$$\underbrace{(2x+3)}_{x=-\frac{3}{2}}\underbrace{(2x-1)}_{x=\frac{1}{2}}=0$$

critical #s

$$4(-3)=-12$$
$$-1+12=11$$
$$-2+6=4$$
$$-3+4=1$$

a-c method

Ex 6: Is $\frac{\pi}{8}$ a critical number for

$$y=2\sin(2x)-2\sqrt{2}\,x \;?$$

Idea: Check $y'(\pi/8)=0$

$$y'=2\cos(2x)(2)-2\sqrt{2}$$
$$=4\cos(2x)-2\sqrt{2}$$

$$y'\left(\frac{\pi}{8}\right) = 4\cos\left(\frac{2\pi}{8}\right) - 2\sqrt{2}$$

$$= 4\cos\left(\frac{\pi}{4}\right) - 2\sqrt{2}$$

$$= 4\,\frac{\sqrt{2}}{2} - 2\sqrt{2}$$

$$= 2\sqrt{2} - 2\sqrt{2} = 0$$

Yes