Lesson 18: Increasing and Decreasing Functions & the First Derivative Test

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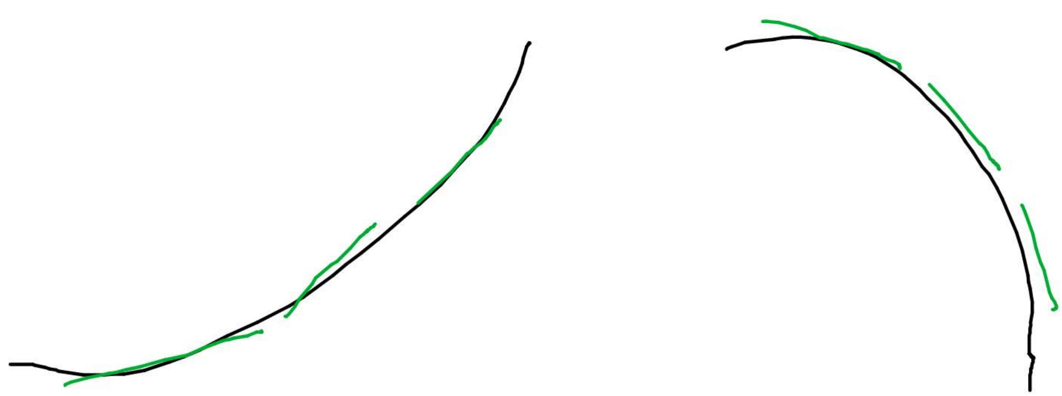
a function is increasing if the function value gets bigger as x gets bigger.

a function is decreasing if the function value gets smaller as x gets bigger.



Notice that slope of the tangent lines

increasing $\Rightarrow$ positive

decreasing $\Rightarrow$ negative

Theorem: Let $f(x)$ be a continuous and differentiable funcition on an open interval, I.

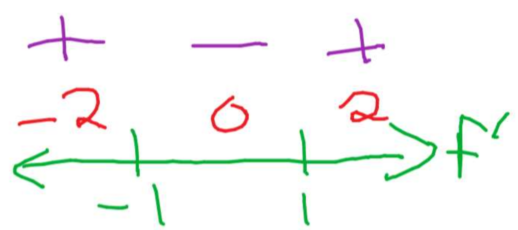
- If $f'(x)>0$ for all x in I, then $f(x)$ is increasing on I.

- If $f'(x)<0$ for all x in I, then $f(x)$ is decreasing on I.

Example 1: Given $f(x)=x^3-3x$. Find where f is inc/dec.

Set your derivative to 0.

$$f'(x)=3x^2-3=0$$
$$3(x^2-1)=0$$
$$3(x-1)(x+1)=0$$
$$x=1,-1$$



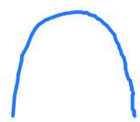
$$f'(x)=3(x-1)(x+1)$$
$$f'(-2)=3\underbrace{(-2-1)}_{-}\underbrace{(-2+1)}_{-}=+$$
$$f'(0)=3\underbrace{(0-1)}_{-}\underbrace{(0+1)}_{+}=-$$
$$f'(2)=3\underbrace{(2-1)}_{+}\underbrace{(2+1)}_{+}=+$$

Inc: $(-\infty,-1)\cup(1,\infty)$

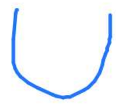
Dec: $(-1,1)$

__The First Derivative Test__: Let c be a critical # of $f(x)$ that is continuous on an open interval, I, containing c.

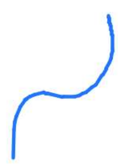
① $\quad + \quad | \quad - \quad f'$ (at c)

ex.  $\Rightarrow$ rel max @ $x=c$

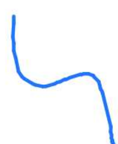
② $\quad - \quad | \quad + \quad f'$ (at c)

ex.  $\Rightarrow$ rel min @ $x=c$

③ $\quad + \quad | \quad + \quad f'$ (at c)

ex.  $\Rightarrow$ neither

④ $\quad - \quad | \quad - \quad f'$ (at c)

ex.  $\Rightarrow$ neither

__Ex 2__: $f(x) = x^3$

ⓐ Find when f is inc/dec.

$f'(x) = 3x^2 = 0$

$x = 0$

$\quad + \quad | \quad + \quad f'$

$-1 \quad 0 \quad 1$

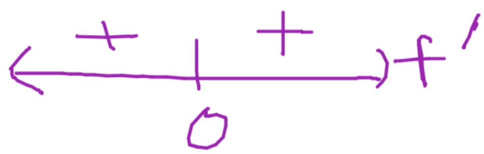
$f'(-1) = 3(-1)^2 > 0$

$f'(1) = 3(1)^2 > 0$

Inc: $(-\infty, \infty)$

Dec: NONE

ⓑ Find relative extrema.



By case 3, there are no relative extrema.

Ex 3: $g(x) = 2x^2 e^{4x+1}$

ⓐ Find when $g(x)$ is inc/dec.

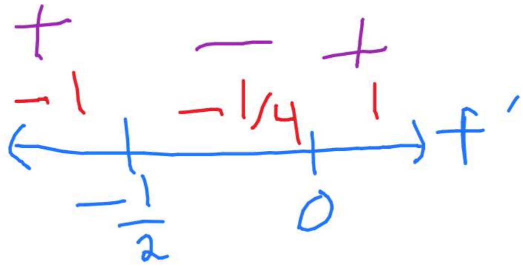
$u(x) = 2x^2 \qquad v(x) = e^{4x+1}$

$u'(x) = 4x \qquad v'(x) = 4e^{4x+1}$

$g'(x) = 4xe^{4x+1} + 2x^2(4e^{4x+1}) = 0$

$$\underbrace{4x}_{x=0}\ \underbrace{e^{4x+1}}_{\text{Nada}}\ \underbrace{[1+2x]}_{x=-1/2} = 0$$

$\boxed{e^x \neq 0}$



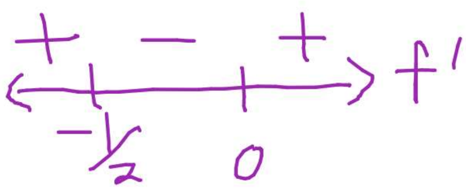
$$f'(x) = 4x e^{4x+1}(1+2x)$$

$$f'(-1) = \underbrace{4(-1)}_{-}\underbrace{e^{4(-1)+1}}_{+}\underbrace{(1+2(-1))}_{-} = +$$

$$f'\left(-\tfrac{1}{4}\right) = \underbrace{4\left(\tfrac{-1}{4}\right)}_{-}\underbrace{e^{4(-1/4)+1}}_{+}\underbrace{\left(1+2\left(\tfrac{-1}{4}\right)\right)}_{+} = -$$

$$f'(1) = \underbrace{4(1)}_{+}\underbrace{e^{4(1)+1}}_{+}\underbrace{(1+2(1))}_{+} = +$$

$$\boxed{\begin{array}{l} e^x \neq 0 \text{ but} \\ \text{moreover} \\ e^x > 0 \end{array}}$$



<u>Inc</u>: $(-\infty, -1/2) \cup (0, \infty)$

<u>Dec</u>: $(-1/2, 0)$

(b) Find relative extremas

Rel max @ $x = -1/2$ (by Case 1)

Rel min @ $x = 0$ (by Case 2)

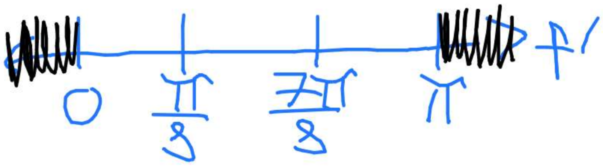
<u>Ex 4</u>: The critical #s of

$$f(x) = 2\sin(2x) - 2\sqrt{2}\,x \quad \text{on } (0, \pi)$$

are $\pi/8$, $7\pi/8$. Does a relative extrema occur @ either pt?

<u>Steps</u>

(1) Find f'

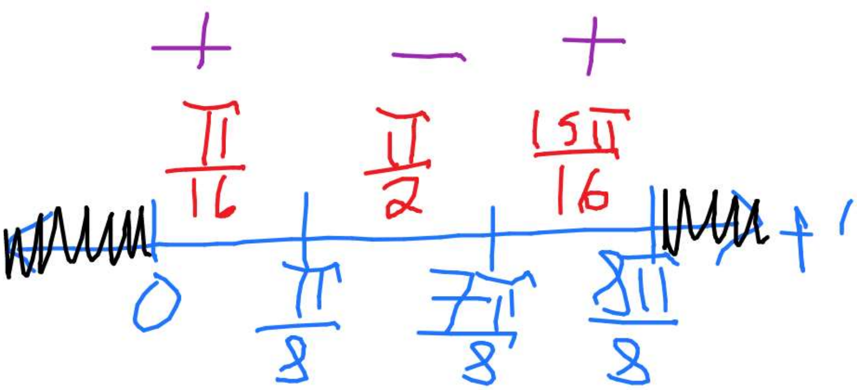


② Test pts
③ Plug those pts into f'
④ Use 1st Derivative Test

$$f(x) = 2\sin(2x) - 2\sqrt{2}\,x$$

$$f'(x) = 2\cos(2x)(2) - 2\sqrt{2}$$

$$= 4\cos(2x) - 2\sqrt{2}$$



$$f'\left(\frac{\pi}{16}\right) > 0$$

$$f'\left(\frac{\pi}{2}\right) < 0$$

$$f'\left(\frac{15\pi}{16}\right) > 0$$

By Case 1, rel max @ $x = \pi/8$
By case 2, rel min @ $x = 7\pi/8$