# Lesson 20: Absolute Extrema on an Interval

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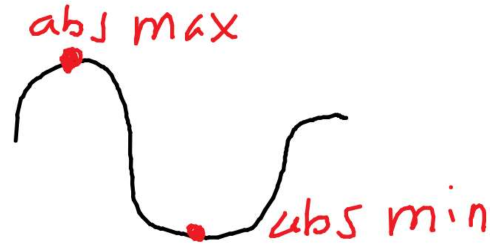
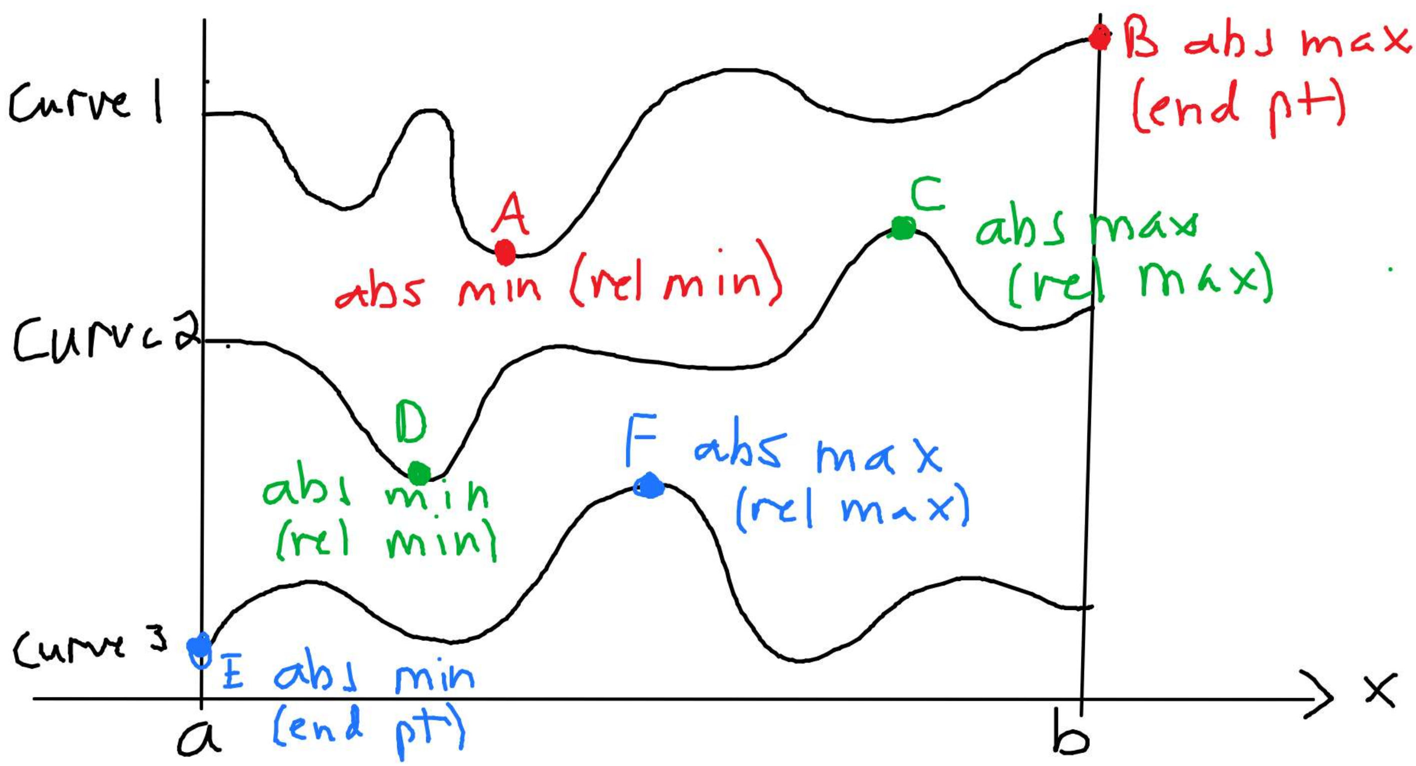
an absolute max is the largest function value on the entire interval.

an absolute min is the smallest function value on the entire interval.



Theorem: If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has both an absolute max and min on the interval.

The abs extrema only occur either @
- critical #s, or
- end pts

Note Relative $\Leftrightarrow$ Local Extremas
Absolute $\Leftrightarrow$ Global Extremas

Steps to find Absolute Extrema

① Find all critical #s. $[f'(x)=0]$

② Plug ① and endpts into $f(x)$.

③ Compare the function values and determine abs extrema.

[i.e. Biggest $f(x)$ value in ② $\Rightarrow$ abs max
Smallest $f(x)$ value in ② $\Rightarrow$ abs min]

Ex 1: Find the abs extrema of

$$y = x^4 - 2x^3 \text{ on } [-1, 1]$$

$$y' = 4x^3 - 6x^2 = 0$$

$$2x^2(2x-3) = 0$$

$$x = 0, \cancel{\tfrac{3}{2}}$$

$x = 0, \cancel{\frac{3}{2}}$

Check x-values are in $[-1, 1]$

| $x$ | $y = x^4 - 2x^3$ | Conclusion |
|---|---|---|
| $-1$ | $1 - 2(-1) = 3$ | Abs max |
| $0$ | $0$ | |
| $1$ | $1 - 2 = -1$ | Abs min |

Ex 2: Find the abs extrema of

$$y = xe^x \quad \text{on } [-2, 0]$$

$u(x) = x \quad v(x) = e^x$

$u'(x) = 1 \quad v'(x) = e^x$

$y' = 1 \cdot e^x + xe^x = 0$

$e^x(1 + x) = 0$

$e^x > 0$ $\quad x = -1$

Check $-1 \in [-2, 0]$ in

| $x$ | $y = xe^x$ | Conclusion |
|---|---|---|
| $-2$ | $-2e^{-2} = -2/e^2$ | Abs min |
| $-1$ | $-e^{-1} = -1/e = -\frac{e}{e^2}$ | |
| $0$ | $0$ | Abs max |

Ex 3: Find the abs extrema of

$$y = -x^2 - 2x \quad \text{on } (-2, 0)$$

$y' = -2x - 2 = 0$

$-2(x + 1) = 0$

$x = -1$

When you have ( ), you need to use 1st or $\boxed{\text{2nd Derivative Test}}$

$y'' = -2$

$y''(-1) = -2 < 0 \Rightarrow$ rel max

$\Rightarrow$ abs max

Ex 3: Find the abs extrema of

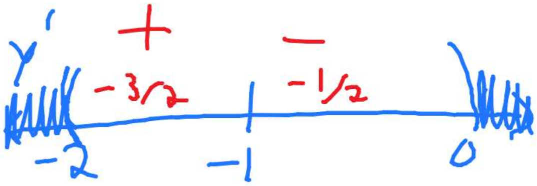
$y = -x^2 - 2x$ on $(-2, 0)$

$y' = -2x - 2 = 0$

$-2(x+1) = 0$

$x = -1$

when you have ( ),
you need to use $\boxed{1^{st}}$
or $2^{nd}$ $\boxed{\text{Derivative Test}}$.




$y'\left(-\frac{3}{2}\right) = -2\left(-\frac{3}{2}+1\right) = - \cdot - = +$

$y'\left(-\frac{1}{2}\right) = -2\left(-\frac{1}{2}+1\right) = - \cdot + = -$

rel max $\Rightarrow$ abs max