Lesson 23: A Summary of Curve Sketching

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We want to graph $f(x) = \frac{x^2-x}{x-3}$.

① Domain: When does f(x) DNE?

Fractions are not defined when deno. = 0.

i.e. $x-3=0 \iff x=3$

Domain: $(-\infty, 3) \cup (3, \infty)$

② x-intercept: Set y = 0. Solve for x.

$\frac{0}{1} = \frac{x^2-x}{x-3}$

$x=0, \quad x-1=0$

$x=1$

$0 = x^2 - x$

$0 = x(x-1)$

x-intercepts: (0,0), (1,0)

③ y-intercept: Set x = 0. Solve for y.

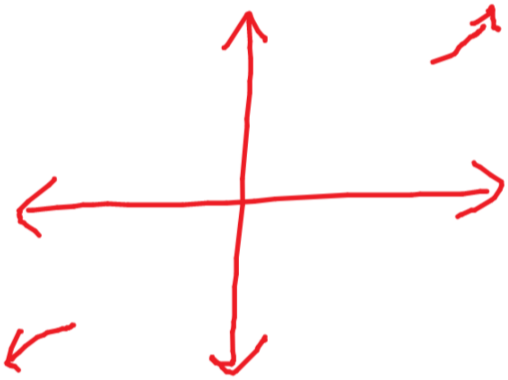
$y = \frac{0^2-0}{0-3} = \frac{0}{-3} = 0$

y-intercept: (0,0)

④ End Behavior: Use the general rule

ⓐ $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2}{x} = \lim_{x\to\infty} x = \infty$

ⓑ $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x = -\infty$



$$f(x) = \frac{x^2 - x}{x-3} = \frac{x(x-1)}{x-3}$$

⑤ Asymptotes

ⓐ Vertical Asymptotes

① Check that f(x) is simplify. ✓

② Set denominator to 0. $x-3=0$

$x=3$

$\boxed{VA \quad x=3}$

ⓑ Horizontal Asymptotes

Use ④. Since $\lim_{x\to\pm\infty} f(x) \neq L$ a constant then there is no HA.

ⓒ Slant Asymptotes

① Check that difference of b/w the exponents of the leading of num. & deno. terms is equal to 1. ✓

② If so, use Sythetic Division or Long Division.

$$\begin{array}{r|rrr} 3 & 1 & -1 & 0 \\ & \downarrow & 3 & 6 \\ \hline & 1 & 2 & 6 \end{array}$$

$$f(x) = \underbrace{x+2} + \frac{6}{x-3}$$

$$\boxed{\begin{array}{l}\text{Slant Asym} \\ y = x+2\end{array}}$$

$$f(x) = \frac{x^2-x}{x-3}$$

⑥ Critical #s: $f'(x) = 0$ & $f'(x)$ DNE

$u(x) = x^2 - x$ $\quad v(x) = x - 3$

$u'(x) = 2x - 1$ $\quad v'(x) = 1$

$$f'(x) = \frac{(2x-1)(x-3) - (x^2-x)}{(x-3)^2}$$

$$= \frac{2x^2 - 7x + 3 - x^2 + x}{(x-3)^2}$$

$$= \frac{x^2-6x+3}{(x-3)^2} = 0$$

	$2x$	-1
x	$2x^2$	$-x$
-3	$-6x$	3

$$\boxed{\begin{array}{c}\text{Recall quadratic} \\ \text{formula} \\ x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}\end{array}}$$

$$f'(x) = \frac{x^2-6x+3}{(x-3)^2} = 0$$

$$x^2 - 6x + 3 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}$$

$$= 3 \pm \sqrt{6}$$

$f'(x)=\frac{x^2-6x+3}{(x-3)^2}$ DNE when $(x-3)^2=0$

$x=3$

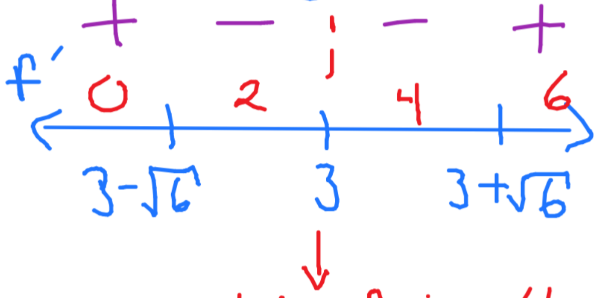
But b/c f(x) DNE @ x=3, x=3 isn't a critical #.

Critical #s: $x=3\pm\sqrt{6}$

$f(x)=\frac{x^2-x}{x-3}$

$f'(x)=\frac{x^2-6x+3}{(x-3)^2}$

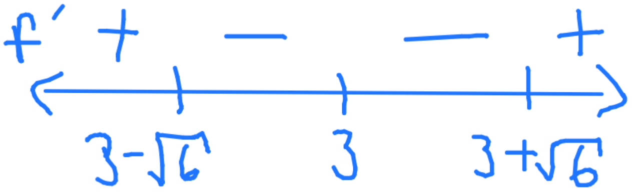
⑦ Increasing & Decreasing



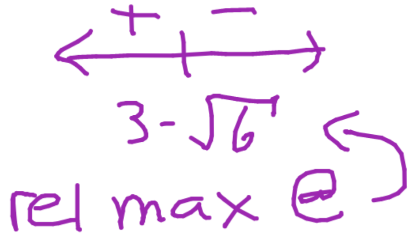
b/c f isn't defined there

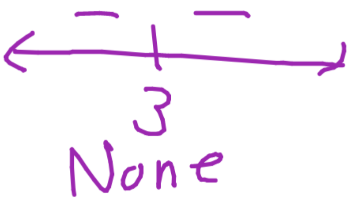
Note deno is always positive. So just plug test pts into f' numerator.

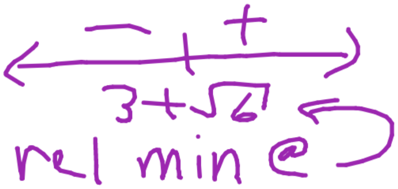
⑧ Relative Max/Min: Use ⑦



By 1st Derivative Test,







$$f(x) = \frac{x^2 - x}{x-3} \qquad f'(x) = \frac{x^2 - 6x + 3}{(x-3)^2}$$

⑨ Concave Up/Down: Find $f''(x) = 0$ or $f''(x)$ DNE

$u(x) = x^2 - 6x + 3 \qquad v(x) = (x-3)^2$

$u'(x) = 2x - 6 \qquad v'(x) = 2(x-3)$

$$f''(x) = \frac{(2x-6)(x-3)^2 - 2(x-3)(x^2-6x+3)}{(x-3)^4}$$

$$= \frac{2\cancel{(x-3)}\left[(x-3)(x-3) - (x^2-6x+3)\right]}{(x-3)^{\cancel{4}\,3}}$$

$$= \frac{2\left[\cancel{x^2} - \cancel{6x} + 9 - \cancel{x^2} + \cancel{6x} - 3\right]}{(x-3)^3}$$

$$f''(x) = \frac{2 \cdot 6}{(x-3)^3} = \frac{12}{(x-3)^3} = 0 \Rightarrow f''(x) \neq 0$$

b/c num. $\neq 0$

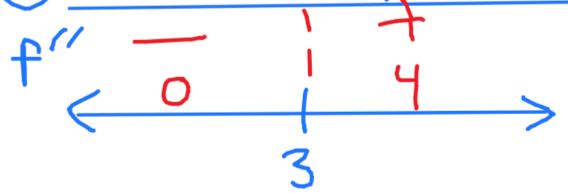
$f''(x)$ DNE when $(x-3)^3=0$

$x=3$

but we know $f(x)$ DNE @ $x=3$

⑨ Concave Up/Down: Find $f''(x)=0$ or $f''(x)$ DNE

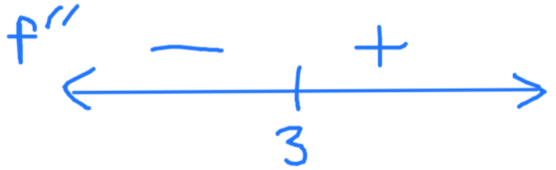
$$f''(x) = \frac{12}{(x-3)^3}$$



Up: $(3,\infty)$

Down: $(-\infty,3)$

⑩ Inflection Pts: Use ⑨ + check for change



Is there a change? Yes

No inflections b/c $f(x)$ isn't defined at $x=3$.

⑪ Graph

$f(x) = \frac{x^2 - x}{x - 3}$