Lessons 24-26: Optimization

Optimization Problems: Often this includes finding the maximum or minimum value of some function.

- e.g. The minimum time to make a certain journey,
- The minimum cost for constructing some object,
- The maximum profit to gain for a business, and so on.

How do we solve an optimization problem?

- Determine a function (**known as objective function)** that we need to maximize or minimize.

- Determine if there are some constraints on the variables. (The equations that describe the constraints are called **the constraint equations**.)
    - If there are constraint equations, rewrite the **objective function** as a function of only one variable.

- Then we can solve for absolute maximum or minimum like we did before.

This space is left for you to take your own notes.

Recipe for Solving an Optimization Problem

Step 1: Identify what quantity you are trying to optimize.

Step 2: Draw a picture (if applicable), corresponding to the problem, and label it with your variables.

Step 3: Express the variable to be optimized as a function of the variables you used in Step 2.

Step 4: Find relations among the variables from Step 2 and express the variable to be optimized a function of just one of the variables from Step 2.

Step 5: Identify the domain for the function you found in Step 4.

Step 6: Find the absolute maximum of the variable to be optimized on this domain.

Step 7: Reread the question and be sure you have answered exactly what was asked.

This space is left for you to take your own notes.

Example 2: Of all the numbers whose sum is 50, find the two that have the maximum product.

Example 3: A carpenter is building a rectangular room with a fixed perimeter of 54 feet. What are the dimensions of the largest room that can be built? What is its area?

Example 4: An open-top box with a square base is to have a volume of 8 cubic feet. Find the dimensions of the box that can be made with the least amount of material.

Example 1: From a thin piece of cardboard 20 in by 20 in, square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume?

HW 24.6: You are designing a paper with an area of 400 cm^2 to contain a printing area in the middle and have the margins of 2 cm at the top and bottom and 4 cm on each side. Find the largest possible printing area.

Example 5: A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area if the total perimeter is 10 feet.

Example 6: A rectangular box is to have a square base and a volume of 800 ft^3. If the material for the base costs $2 per square foot, the material for the sides costs $4 per square foot, and the material for the top costs $1 per square foot, determine the minimum cost for constructing such a box.

Example 7: A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 2400 - 200p$ units. Each unit costs $5 to make.

1. What price, p, should the company charge to maximize their revenue?

Example 7: A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 2400 - 200p$ units. Each unit costs \$5 to make.

2. What price, p, should the company charge to maximize their profit?

<u>Example 8:</u> Find the point on the graph of $f(x) = 2x + 4$ that is the closest to the point $(1,3)$.

HW 25.4: For a cylinder with a surface area of 20, what is the maximum volume that it can have? Recall that the volume of a cylinder is $\pi r^2 h$ and the surface area is $2\pi rh + 2\pi r^2$ where r is the radius and h is the height.

HW 26.1: A rectangular recreational field needs to be built outside of a gymnasium. Three walls of fencing are needed and the fourth wall is to be a wall of the gymnasium itself. The ideal area for such a field is exactly $490000 ft^2$. In order to minimize costs, it is necessary to construct the fencing using the least amount of material possible. Assuming that the material used in the fencing costs $25/ft, what is the least amount of money needed to build this fence of ideal area?