Lesson 27: Antiderivatives and Indefinite Integration

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The reverse process of differentiation is called $\boxed{\text{antidifferentiation}}$

Ex. Suppose $f(x) = 3x$. Whose function, F, has a derivative $3x$?

$$F(x) = \frac{3}{2}x^2$$

Check $f(x) = F'(x)$.

$$F'(x) = \frac{3}{2}(2)x = 3x = f(x)$$

Def: A function $F(x)$ is an antiderivative of $f(x)$ over an interval if

$F'(x) = f(x)$ for every x in the interval.

So $F(x) = \frac{3}{2}x^2$ is an antiderivative of

$f(x) = 3x$

Question: Are $G(x)=\frac{3}{2}x^2+10.25$

$H(x)=\frac{3}{2}x^2+10e$

$J(x)=\frac{3}{2}x^2-100000$

antiderivatives of $f(x)$ as well?

i.e. $G'(x)=f(x)$

$H'(x)=f(x)$

$J'(x)=f(x)$

Yes

Theorem: If $F(x)$ is an antiderivative of $f(x)$ over an interval, then $G(x)$ is also an antiderivative of $f(x)$ over this interval if and only if $F(x)=G(x)+c$ where c is a constant

Why? $(F(x))'=(G(x)+c)'$

$F'(x)=G'(x)$

$f(x)=F'(x) \implies f(x)=G'(x)$

Process of finding all the antiderivatives of a function is called $\boxed{\text{indefinite integration.}}$

Denoted by $\underbrace{\int f(x)\,dx}= F(x) + C$ where C is a constant

Read as "integral of $f(x)$"

- $\int$ integral sign
- $f(x)$ integrand
- x integration variable
- C constant of integration

Differentiation Rule	Integration Rule
$\frac{d}{dx}(c) = 0$	$\int 0\,dx = C$
$\frac{d}{dx}(kx) = k$	$\int k\,dx = kx + C$
$\frac{d}{dx}(kf(x)) = kf'(x)$	$\int kf'(x)\,dx = k\int f'(x)\,dx = kf(x) + C$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(x^{n+1}) = (n+1)x^n$

$\int nx^{n-1}\,dx = x^n + C$

$\int (n+1)x^n\,dx = x^{n+1} + C$

$(n+1)\int x^n\,dx = x^{n+1} + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Ex 1: Find the indefinite integral

$$\int (x^2 + 2\sqrt{x})\,dx$$

$$\int (x^2 + 2x^{1/2})\,dx = \frac{x^{2+1}}{2+1} + 2\,\frac{x^{1/2+1}}{1/2+1} + C$$

$$= \frac{x^3}{3} + 2\,\frac{x^{3/2}}{3/2} + C$$

$$= \frac{x^3}{3} + 2 \cdot \frac{2}{3} x^{3/2} + C$$

$$= \frac{x^3}{3} + \frac{4}{3} x^{3/2} + C$$

Differentiation Rule	Integration Rule
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + c$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + c$

Differentiation Rule	Integration Rule
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x \, dx = -\csc x + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\ln x) = \frac{1}{x}, \; x > 0$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$

Always take the derivative of your answer!!!

Especially when you have trig functions!!!

Ex 2: Evaluate $\int \frac{\sin x + \cos x}{2} dx$

$$\frac{1}{2}\int (\sin x + \cos x) dx = \frac{1}{2}\left[\underbrace{\int \sin x \, dx}_{-\cos x} + \underbrace{\int \cos dx}_{\sin x}\right]$$

$$= \frac{1}{2}(-\cos x + \sin x) + C$$

$$= -\frac{1}{2}\cos x + \frac{1}{2}\sin x + C$$