Lesson 2: Finding Limits Numerically

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Def: If $f(x)$ approaches ($\to$) L as $x \to c$, we say that the limit of $f(x)$ as $x \to c$ is L.

$$\lim_{x \to c} f(x) = L$$

Note that f does not need to be defined @ $x = c$ for the limit to exist.

Def of Infinite Limits

- If $f(x)$ inc or dec w/o bound as $x \to c$, then $\lim_{x \to c} f(x)$ is an infinite limit.

If $f(x)$ inc w/o bound,

$$\lim_{x \to c} f(x) = +\infty$$

If $f(x)$ dec w/o bound,

$$\lim_{x \to c} f(x) = -\infty$$

Finding Limits Numerically

we evaluate $f(x)$ at values of x that are getting closer and closer to c and see what happens to $f(x)$.

Ex 1: Evaluate $\lim_{x \to 4} \underbrace{(2x-3)}_{f(x)}$ numerically

x	3.9	3.99	3.999	3.9999	4
$f(x)$	4.8	4.98	4.998	4.9998	—

x	4.0001	4.001	4.01	4.1
$f(x)$	5.0002	5.002	5.02	5.2

$$\boxed{\lim_{x \to 4} (2x-4 = 5}$$

Ex 2: Evaluate $\lim_{x \to 3} \frac{x^3-3x^2}{x-3}$ numerically $\to f(x)$

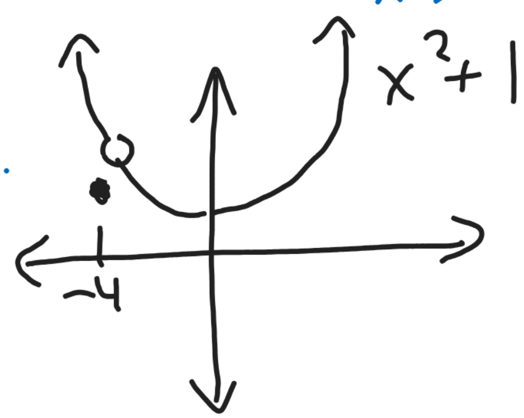
x	2.99	2.999	2.9999	3
$f(x)$	8.941	8.9940	8.9994	—

x	3.001	3.01	3.1
$f(x)$	9.006	9.0601	9.61

$$\boxed{\lim_{x \to 3} \frac{x^3-3x^2}{x-3} = 9}$$

Ex 3: Given $f(x) = \begin{cases} x^2+1 & \text{if } x \neq -4 \\ 2 & \text{if } x = -4 \end{cases}$

Evaluate $\lim_{x \to -4} f(x)$.



x	-4.001	-4.0001	-4
$f(x)$	17.008	17.0008	—

x	-3.9999	-3.999
$f(x)$	16.9992	16.992

$\lim_{x \to -4} f(x) = 17$

Question: Is $17 = f(-4)$?

NO!!! $\quad f(-4) = 2$

Moral: $\lim_{x \to c} f(x)$ doesn't necessarily equal $f(c)$.

One-Sided Limits

a one-sided limit is the value that the function on $f(x) \to L$ as $x \to c$ from the left or right.

If $f(x) \to L$ as $x \to c$ from the left then $\lim_{x \to c^-} f(x) = L$ (Left-sided Limit)

If $f(x) \to L$ as $x \to c$ from the right then $\lim_{x \to c^+} f(x) = L$. (Right-sided Limit)

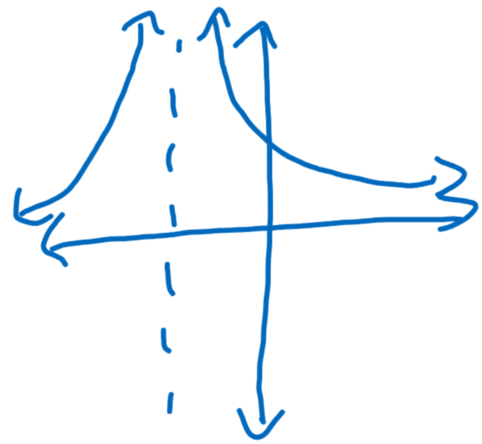
Ex 4:

a) $\lim_{x \to 1^-} \frac{1}{(x+1)^2} = \infty$

b) $\lim_{x \to 1^+} \frac{1}{(x+1)^2} = \infty$

c) $\lim_{x \to 1} \frac{1}{(x+1)^2} = \infty$

d) $f(1)$ undefined



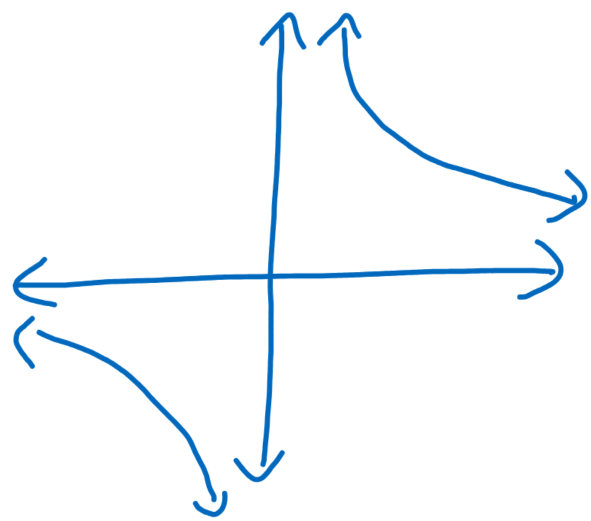
Ex 5:

a) $\lim_{x \to 0^-} \frac{1}{x} = -\infty$

b) $\lim_{x \to 0^+} \frac{1}{x} = \infty$

c) $\lim_{x \to 0} \frac{1}{x}$ DNE

d) $f(0)$ undefined



Moral If $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$, then

$\lim_{x \to c} f(x)$ DNE