Lesson 30 Definite Integrals

Lesson 30: Definite Integrals

When the # of rectangles used gets bigger and bigger, the approx gets better and better.

i.e. the approx gets closer and closer to the exact signed area

What happens when $n \to \infty$?

Left/Right Riemann Sum approaches the actual signed area.

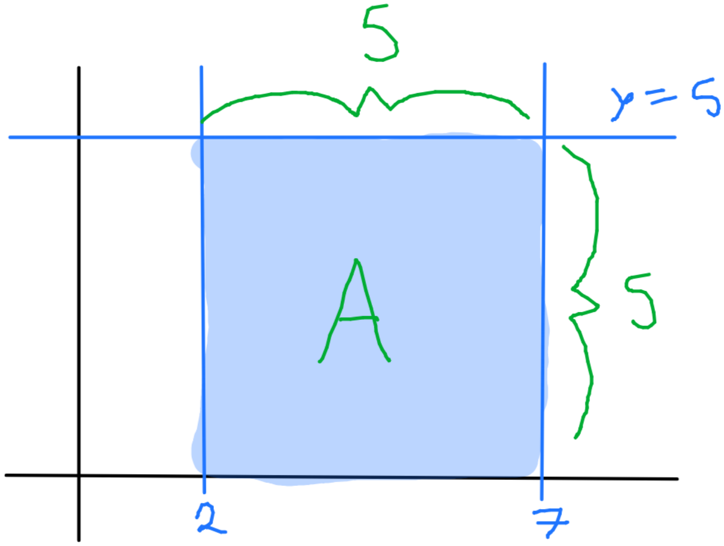
$$\text{Signed Area} = \int_a^b f(x)\,dx$$

a – lower limit of integration
b – upper limit

An integral w/ lower and upper limits is called a definite integral.

So no more +C, when you see lower/upper limits (i.e. a & b)

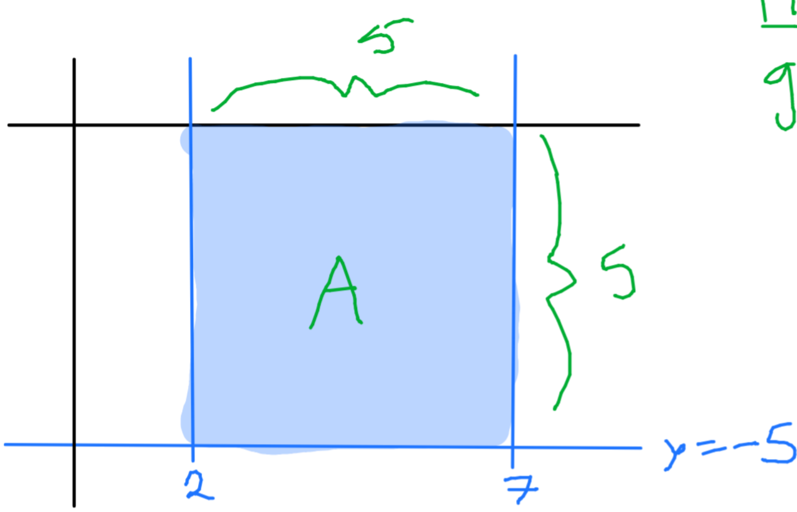
Ex 1: Evaluate the definite integral $\int_2^7 5\,dx$ by using geometric formulas.



$$A = 5(5) = 25$$

$$\int_2^7 5\,dx = 25$$

HW 30.2 Evaluate the definite integral $\int_2^7 -5\,dx$ by using geometric formulas.



Method 1: Use geometric formulas

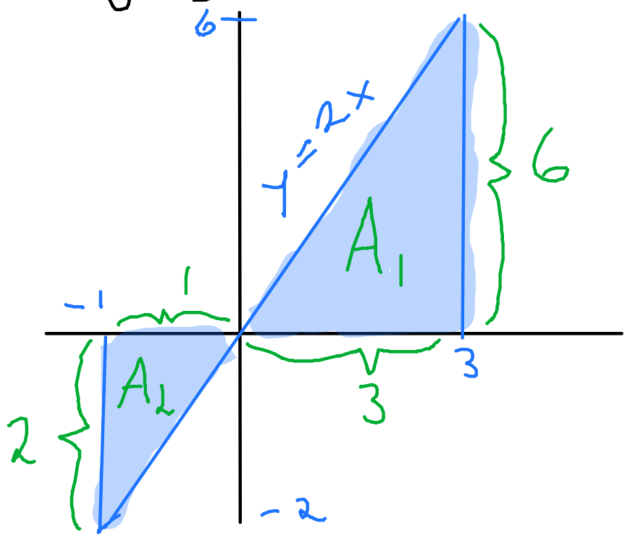
$$A = 25$$

$$\int_2^7 -5\,dx = -25$$

Method 2: Abusing your answer from Ex 1

$$\int_2^7 -5\,dx = -\underbrace{\int_2^7 5\,dx}_{25} = -25$$

Ex 2: Evaluate the definite integral $\int_{-1}^{3} 2x\,dx$ by using geometric formulas.

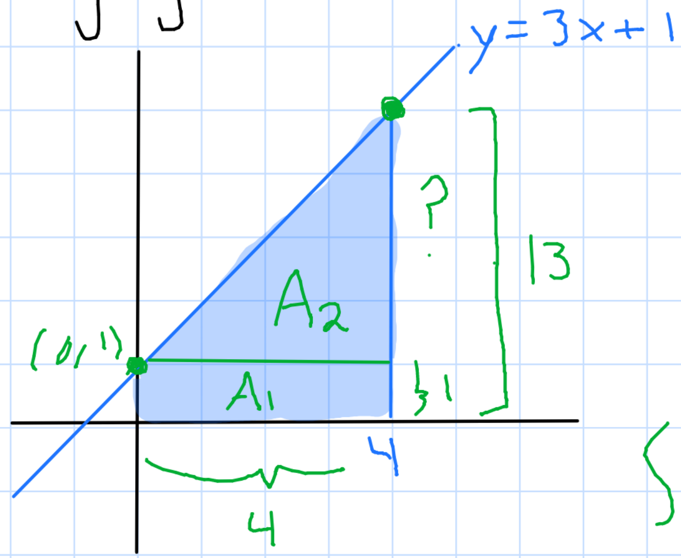


$$A_1 = \frac{1}{\cancel{2}}(3)(\cancel{6}^{3}) = 9$$

$$A_2 = \frac{1}{\cancel{2}}(1)(\cancel{2}) = 1$$

$$\int_{-1}^{3} 2x\,dx = A_1 - A_2$$
$$= 9 - 1$$
$$= 8$$

Ex 3: Evaluate the definite integral $\int_{0}^{4} (3x+1)\,dx$ by using geometric formulas.



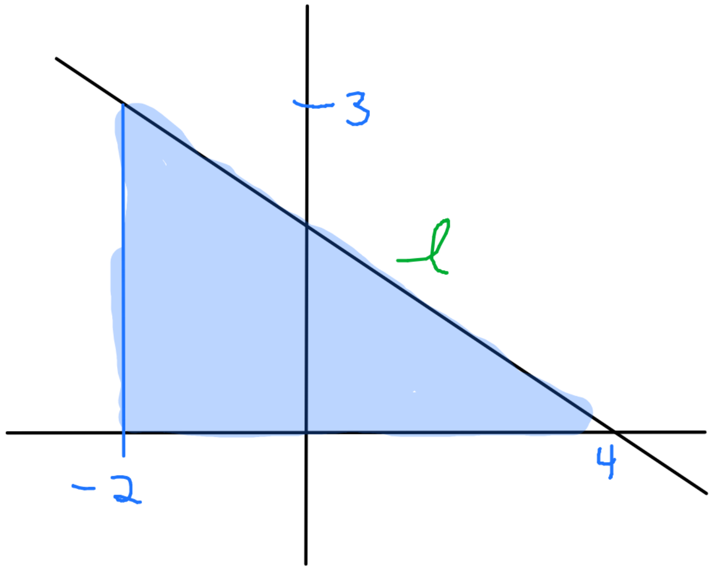
$$A_1 = 1(4) = 4$$

$$y = 3(4) + 1 = 13$$
$$? = 13 - 1 = 12$$

$$A_2 = \frac{1}{2}(4)(12) = 24$$

$$\int_{0}^{4} (3x+1)\,dx = 24 + 4 = 28$$

HW 30.6: Write the definite integral that represents the shaded area below.



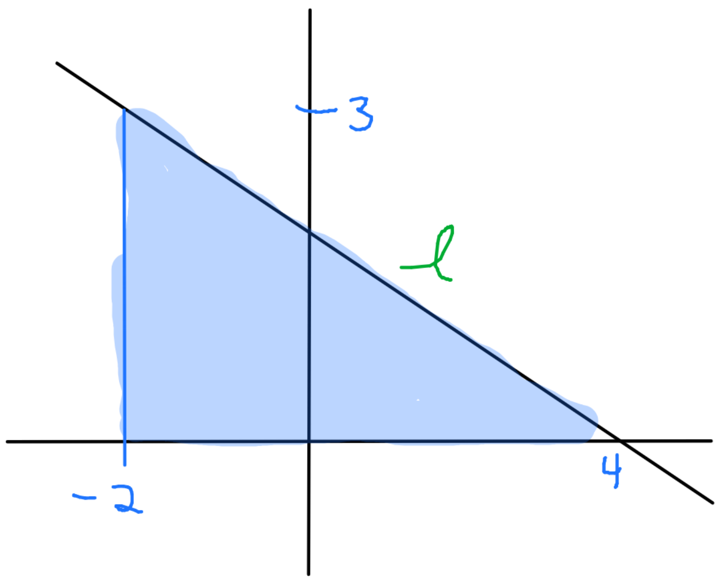
$$\int_{-2}^{4} \boxed{} \, dx$$

$(-2, 3)$ & $(4, 0)$

$$m = \frac{0-3}{4-(-2)} = \frac{-3}{6} = \frac{-1}{2}$$

$$y = -\frac{1}{2}x + b$$

HW 30.6: Write the definite integral that represents the shaded area below.



$$\boxed{\int_{-2}^{4} \left(-\frac{1}{2}x + 2\right) dx}$$

$(-2, 3)$ & $\boxed{(4, 0)}$

$$y = -\frac{1}{2}x + b$$

$$0 = -\frac{1}{2}(4) + b$$

$$0 = -2 + b$$

$$b = 2$$