Lesson 32: The Fundamental Theorem of Calculus (FTC)

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Recall

① If we want to find all the antiderivatives of $f(x)$, we evaluate $\int f(x)\,dx$.

② If we want to find signed area under a curve of $f(x)$ from a to b, we evaluate $\int_a^b f(x)\,dx$.

Note both have an integral sign. Can we connect ① and ②?

Fundamental Theorem of Calculus (FTC)

Suppose $f(x)$ is continuous on the interval $[a,b]$. If $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

In practice, to integrate $\int_a^b f(x)\,dx$, we write

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$$

We read ↑ as "$F(x)$ evaluated from a to b"

Ex 1: Evaluate $\int_0^3 2x\,dx$.

$$\int_0^3 2x\,dx = \frac{2x^2}{2}\Big]_0^3 = x^2\Big]_0^3$$
$$= 3^2 - 0^2$$
$$= 9 - 0$$
$$= 9$$

Recall
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \cancel{+C}$$
we can ignore C b/c we have limits

Ex 2: Evaluate $\int_0^{\pi/4} \sec^2 x\,dx$

$$\int_0^{\pi/4} \sec^2 x\,dx = \tan x\Big]_0^{\pi/4}$$
$$= \tan\left(\frac{\pi}{4}\right) - \tan(0)$$
$$= 1 - 0$$
$$= 1$$

Recall
$$\int \sec^2 x\,dx = \tan x \cancel{+C}$$
we can ignore C b/c we have limits

Ex 3: Evaluate $\int_4^9 \frac{2x^3+\sqrt{x}}{x^2}\,dx$

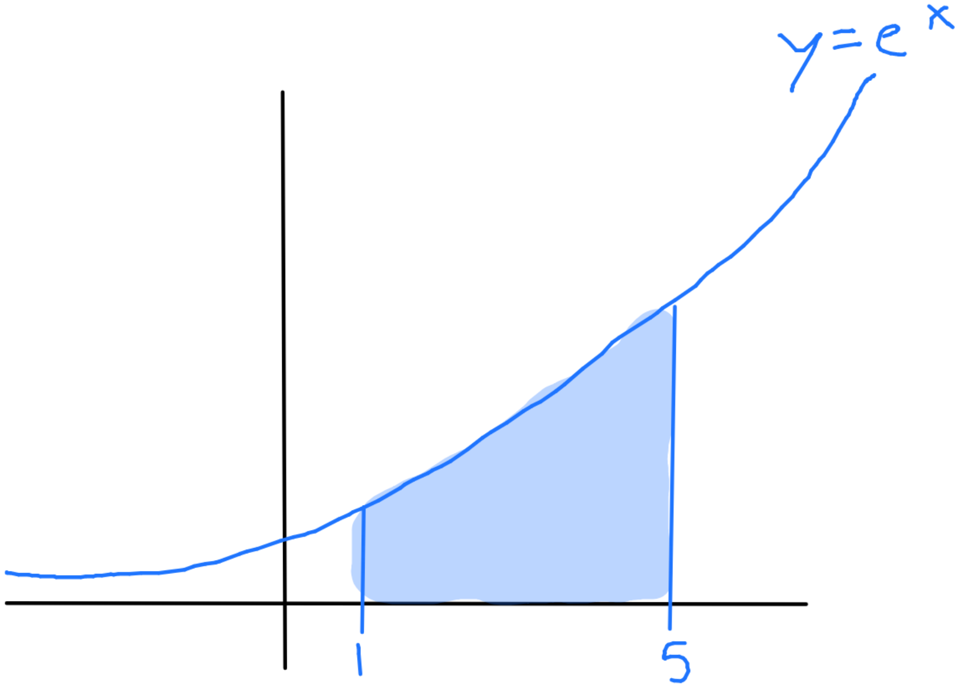
Recall
$\int x^n dx = \frac{x^{n+1}}{n+1} \cancel{+C}$

we can ignore C
b/c we have limits

$$\int_4^9 \frac{2x^3+\sqrt{x}}{x^2}\,dx = \int_4^9 \frac{2x^3+x^{1/2}}{x^2}\,dx$$
$$= \int_4^9 \frac{2x^3}{x^2} + \frac{x^{1/2}}{x^2}\,dx$$
$$= \int_4^9 2x + x^{-3/2}\,dx$$
$$= \left(\frac{2x^2}{2} + \frac{x^{-1/2}}{-1/2}\right)\Big]_4^9$$
$$= \left(x^2 - 2x^{-1/2}\right)\Big]_4^9$$
$$= \left(x^2 - \frac{2}{\sqrt{x}}\right)\Big]_4^9$$
$$= \left(9^2 - \frac{2}{\sqrt{9}}\right) - \left(4^2 - \frac{2}{\sqrt{4}}\right)$$
$$= 81 - \frac{2}{3} - 16 + 1 = \frac{196}{3}$$

Ex 4: Find the area of the region bounded by the graphs of the following equations.

$y = e^x$, $y = 0$, $x = 1$, and $x = 5$



$$\int_1^5 e^x\,dx$$

$$= e^x\Big]_1^5$$

$$= \boxed{e^5 - e^1}$$