# Lesson 33: The Fundamental Theorem of Calculus (FTC) Part 2

Lesson 33: The Fundamental Theorem of Calculus (FTC) Pt 2

Recall $\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$

Question: $\int_a^b g'(x)\,dx = ?$

Well the antiderivative of $g'(x)$ is $g(x)$.

So $\int_a^b g'(x)\,dx = g(x)\Big]_a^b = g(b) - g(a)$

Ex 5: The growth rate of the population of a city is

$$p'(t) = -500(3-t)$$

where t is time in years. How does the population change t = 1 year to t = 3 years?

$$\int_1^3 p'(t)\,dt = \int_1^3 -500(3-t)\,dt$$

$$= -500\int_1^3 (3-t)\,dt$$

$$= -500\left[3t - \frac{t^2}{2}\right]_1^3$$

$$= -500\left[\left(3(3) - \frac{3^2}{2}\right) - \left(3(1) - \frac{1^2}{2}\right)\right]$$

$$= -500\left[9 - \frac{9}{2} - 3 + \frac{1}{2}\right]$$

$= -500\left(6 - \frac{8}{2}\right)$

$= -500(6-4)$

$= -1000$

Recall

- Displacement is the difference in position.
- It could be positive or negative
- The sign indicates the direction.

By FTC,

$$\int_a^b v(t)\,dt = \int_a^b s'(t)\,dt = s(t)\Big]_a^b$$

$$= s(b) - s(a)$$

Ex 6: The velocity function, in feet per second, is given for a particle moving along a straight line

$$v(t) = -10t + 20$$

where t is in seconds.

(a) Find the displacement from t = 0 to t = 2 seconds.

$$\int_0^2 v(t)\,dt = \int_0^2 [-10t + 20]\,dt$$

$$= \left(-\frac{10t^2}{2} + 20t\right)\Big]_0^2$$

$$= (-5t^2 + 20t)\Big]_0^2$$

$$= (-5(2)^2 + 20(2)) - \cancel{(-5(0)^2 + 20(0))} \to 0$$

$$= -20 + 40 = 20$$

Ex 6: The velocity function, in feet per second, is given for a particle moving along a straight line

$$v(t) = -10t + 20$$

where t is in seconds.

(b) Find the displacement from t = 0 to t = 4 seconds.

From (a)

$$\int_0^4 v(t)\,dt \overset{\uparrow}{=} \left(-5t^2 + 20t\right)\Big]_0^4$$

$$= \left(-5(4)^2 + 20(4)\right) - \cancel{\left(-5(0)^2 + 20(0)\right)} \to 0$$

$$= -80 + 80$$

$$= \boxed{0}$$

HW 33.4: A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where t is time in hours after 9:00 am and the rate r(t) is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

9:00am $\Rightarrow t = 0$

10:00am $\Rightarrow t = 1$

1:00pm $\Rightarrow t = 4$

$$\int_1^4 6\sqrt{t}\,dt = \int_1^4 6t^{1/2}\,dt$$

$$= 6\,\frac{t^{3/2}}{3/2}\Big]_1^4$$

$$= 6\left(\frac{2}{3}\right)t^{3/2}\Big]_1^4$$

$$= 4t^{3/2}\Big]_1^4$$

$$= 4(4)^{3/2} - 4(1)^{3/2}$$

$$= 4\cdot 2^3 - 4(1)$$

$$= 32 - 4$$

$$= \boxed{28}$$

HW 33.4: A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where t is time in hours after 9:00 am and the rate r(t) is in cubic feet per hour.

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

Solve $\int_0^x 6t^{1/2}\,dt = 121$ for $x$.

$$121 = \int_0^x 6t^{1/2}\,dt \overset{\text{by (a)}}{=} 4t^{3/2}\Big]_0^x = 4x^{3/2} - 4(0)^{3/2}$$

$$121 = 4x^{3/2}$$

$$x = \boxed{\left(\tfrac{121}{4}\right)^{2/3}}$$