Lesson 3: Finding Limits Graphically

Graphically, we will look at the portion of the curve of $f(x)$ near $x=c$ and see what the function value, y, approaches as x gets closer to c from the left or the right respectively.

If $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$, then

$$\boxed{\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = \lim_{x \to c} f(x)} \quad (*)$$

Note this doesn't imply $(*) = f(c)$.

Find the following limits:

Ex 2

(a) $\lim_{x \to -5^-} f(x) = 2$

(c) $\lim_{x \to -5} f(x) = \text{DNE}$

(b) $\lim_{x \to -5^+} f(x) = 3$

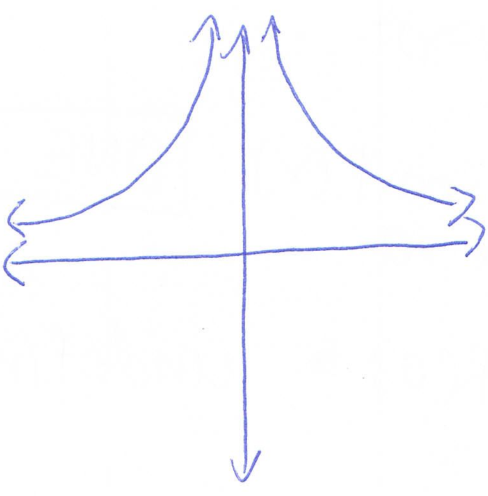
(d) $\lim_{x \to 1^-} f(x) = 3$

(f) $\lim_{x \to 1} f(x) = 3$

(e) $\lim_{x \to 1^+} f(x) = 3$

Ex 3:

$y = \frac{1}{x^2}$



(a) $\lim_{x \to 0^-} f(x) = \infty$

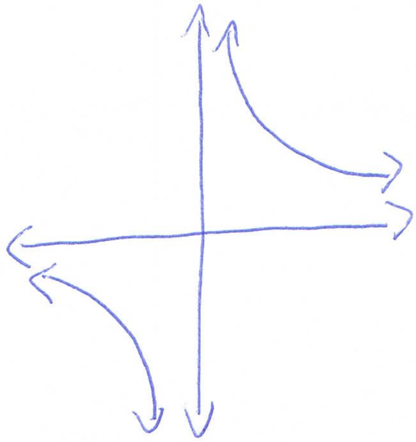
(b) $\lim_{x \to 0^+} f(x) = \infty$

(c) $\lim_{x \to 0} f(x) = \infty$

(d) $f(0)$ undefined

Ex 4:

$y = \frac{1}{x}$



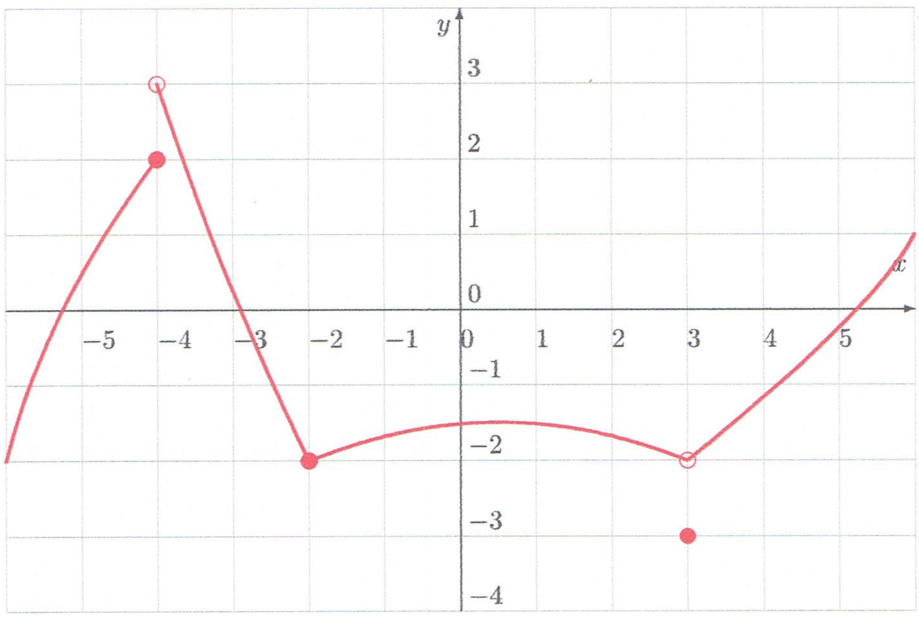
ⓐ $\lim_{x \to 0^-} f(x) = -\infty$

ⓑ $\lim_{x \to 0^+} f(x) = \infty$

ⓒ $\lim_{x \to 0} f(x)$ **DNE**

The remainder of the problems are left for those who want extra practice.

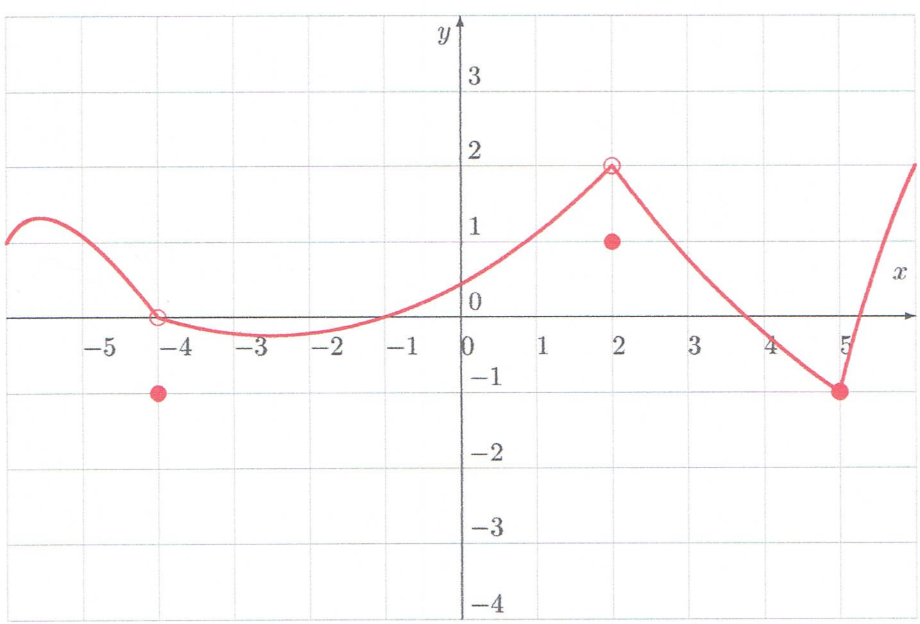
3. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to -2^-} f(x)$ *b)* $\lim_{x\to -2^+} f(x)$ *c)* $\lim_{x\to -2} f(x)$ *d)* $\lim_{x\to -4} f(x)$ *e)* $\lim_{x\to 3} f(x)$

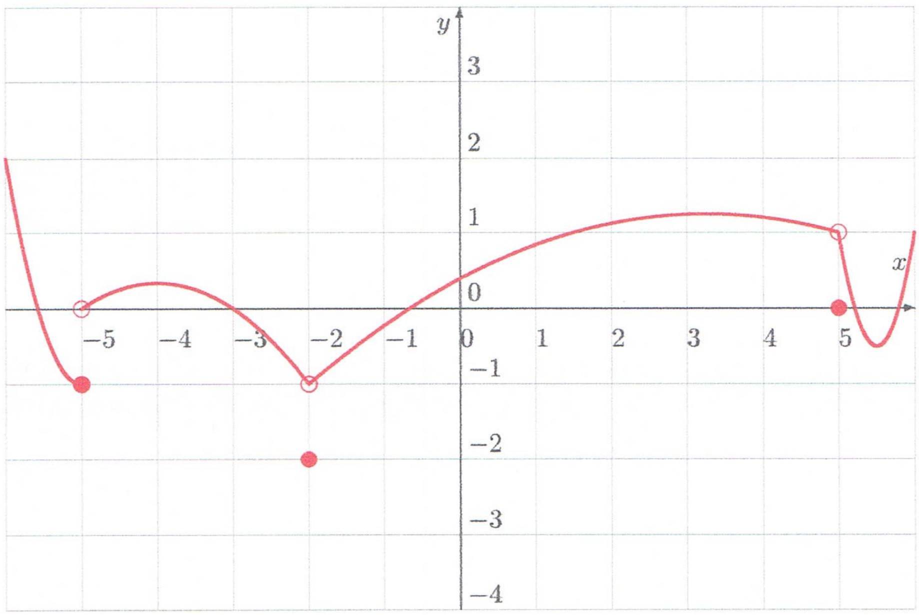
4. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to 2^-} f(x)$ *b)* $\lim_{x\to 2^+} f(x)$ *c)* $\lim_{x\to 2} f(x)$ *d)* $\lim_{x\to -4} f(x)$ *e)* $\lim_{x\to 5} f(x)$

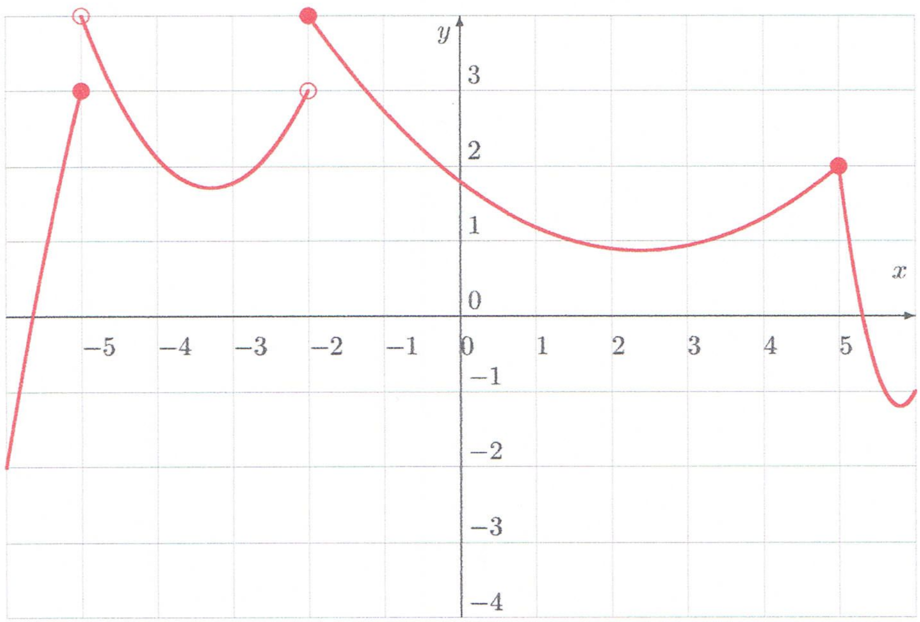
5. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to -2^-} f(x)$ *b)* $\lim_{x\to -2^+} f(x)$ *c)* $\lim_{x\to -2} f(x)$ *d)* $\lim_{x\to -5} f(x)$ *e)* $\lim_{x\to 5} f(x)$

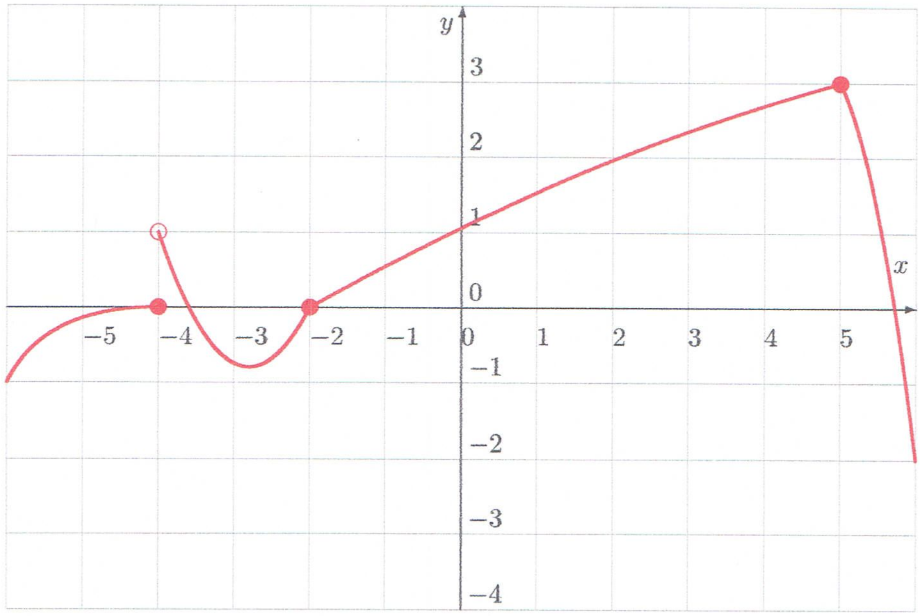
6. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to -2^-} f(x)$ *b)* $\lim_{x\to -2^+} f(x)$ *c)* $\lim_{x\to -2} f(x)$ *d)* $\lim_{x\to -5} f(x)$ *e)* $\lim_{x\to 5} f(x)$

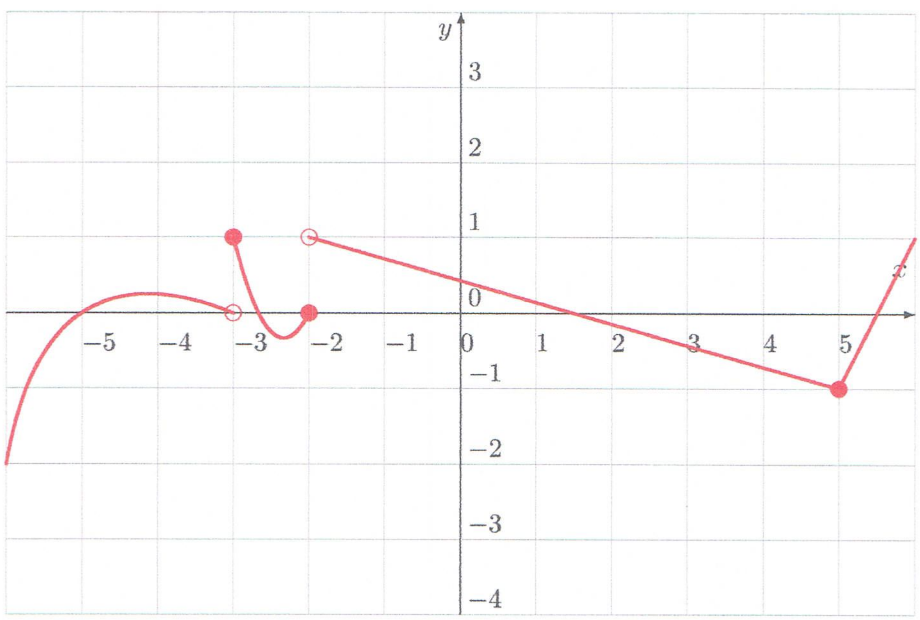
7. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to -2^-} f(x)$ b) $\lim_{x\to -2^+} f(x)$ c) $\lim_{x\to -2} f(x)$ d) $\lim_{x\to -4} f(x)$ e) $\lim_{x\to 5} f(x)$

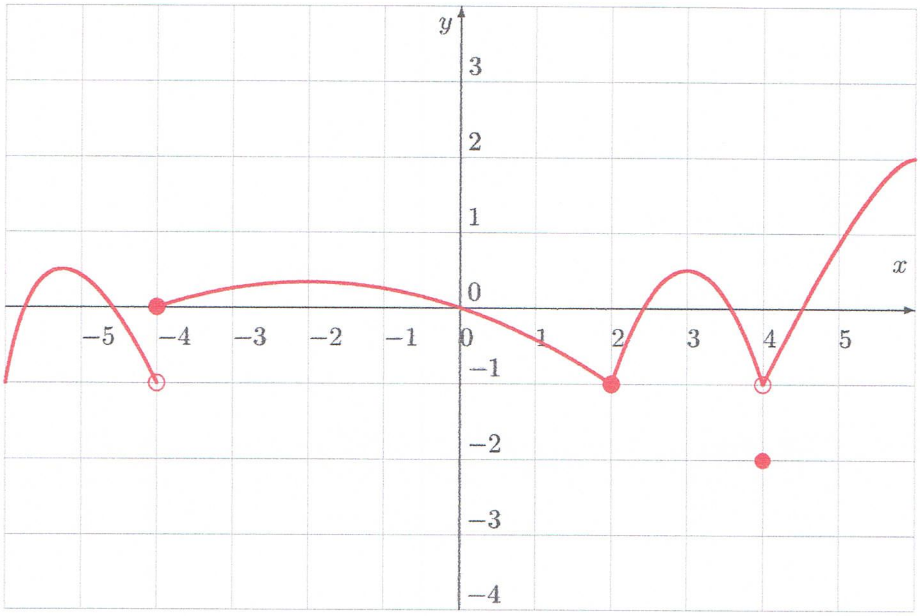
8. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x\to -2^-} f(x)$ b) $\lim_{x\to -2^+} f(x)$ c) $\lim_{x\to -2} f(x)$ d) $\lim_{x\to -3} f(x)$ e) $\lim_{x\to 5} f(x)$

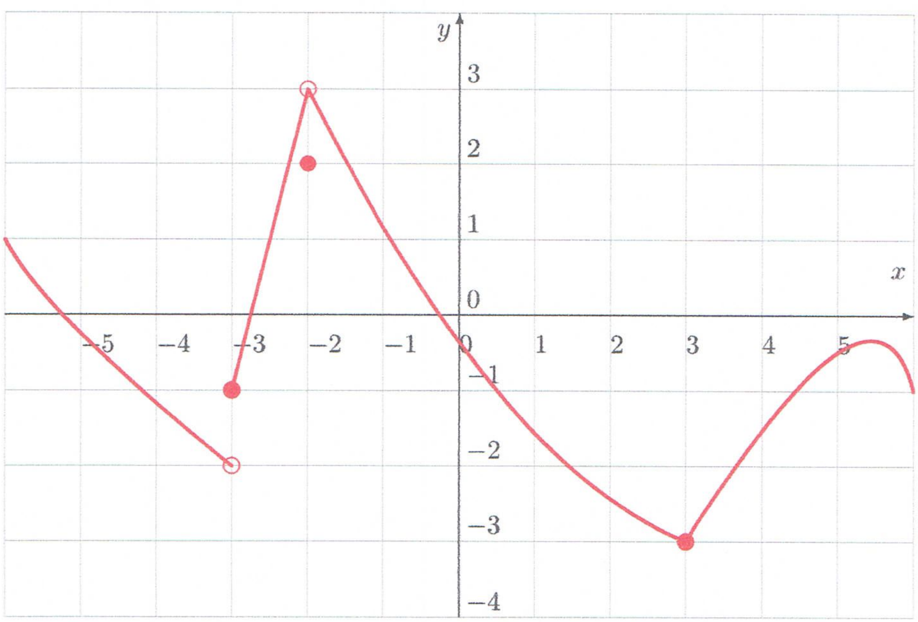
9. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x \to 2^-} f(x)$ b) $\lim_{x \to 2^+} f(x)$ c) $\lim_{x \to 2} f(x)$ d) $\lim_{x \to -4} f(x)$ e) $\lim_{x \to 4} f(x)$

10. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x \to -2^-} f(x)$ b) $\lim_{x \to -2^+} f(x)$ c) $\lim_{x \to -2} f(x)$ d) $\lim_{x \to -3} f(x)$ e) $\lim_{x \to 3} f(x)$

Answers: 1. $a)$ 2 $b)$ 2 $c)$ 2 $d)$ DNE $e)$ DNE
2. $a)$ 3 $b)$ 3 $c)$ 3 $d)$ DNE $e)$ DNE
3. $a)$ -2 $b)$ -2 $c)$ -2 $d)$ DNE $e)$ -2
4. $a)$ 2 $b)$ 2 $c)$ 2 $d)$ 0 $e)$ -1
5. $a)$ -1 $b)$ -1 $c)$ -1 $d)$ DNE $e)$ 1
6. $a)$ 3 $b)$ 4 $c)$ DNE $d)$ DNE $e)$ 2
7. $a)$ 0 $b)$ 0 $c)$ 0 $d)$ DNE $e)$ 3
8. $a)$ 0 $b)$ 1 $c)$ DNE $d)$ DNE $e)$ -1
9. $a)$ -1 $b)$ -1 $c)$ -1 $d)$ DNE $e)$ -1
10. $a)$ 3 $b)$ 3 $c)$ 3 $d)$ DNE $e)$ -3

Solutions:

1.

a) $\lim_{x\to -1^-} f(x) = 2$

b) $\lim_{x\to -1^+} f(x) = 2$

c) $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$. Therefore $\lim_{x\to -1} f(x) = 2$

d) $\lim_{x\to -4^-} f(x) \neq \lim_{x\to -4^+} f(x)$. Therefore $\lim_{x\to -4} f(x) = \text{DNE}$

e) $\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$. Therefore $\lim_{x\to 4} f(x) = \text{DNE}$

2.

a) $\lim_{x\to 1^-} f(x) = 3$

b) $\lim_{x\to 1^+} f(x) = 3$

c) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$. Therefore $\lim_{x\to 1} f(x) = 3$

d) $\lim_{x\to -5^-} f(x) \neq \lim_{x\to -5^+} f(x)$. Therefore $\lim_{x\to -5} f(x) = \text{DNE}$

e) $\lim_{x\to 5^-} f(x) \neq \lim_{x\to 5^+} f(x)$. Therefore $\lim_{x\to 5} f(x) = \text{DNE}$

3.

a) $\lim_{x\to -2^-} f(x) = -2$

b) $\lim_{x\to -2^+} f(x) = -2$

c) $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$. Therefore $\lim_{x\to -2} f(x) = -2$

d) $\lim_{x\to -4^-} f(x) \neq \lim_{x\to -4^+} f(x)$. Therefore $\lim_{x\to -4} f(x) = \text{DNE}$

e) $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$. Therefore $\lim_{x\to 3} f(x) = -2$

4.

a) $\lim_{x\to 2^-} f(x) = 2$

b) $\lim_{x\to 2^+} f(x) = 2$

c) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$. Therefore $\lim_{x\to 2} f(x) = 2$

d) $\lim_{x\to -4^-} f(x) = \lim_{x\to -4^+} f(x)$. Therefore $\lim_{x\to -4} f(x) = 0$

e) $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$. Therefore $\lim_{x\to 5} f(x) = -1$

5.

a) $\lim_{x\to -2^-} f(x) = -1$

b) $\lim_{x\to -2^+} f(x) = -1$

c) $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$. Therefore $\lim_{x\to -2} f(x) = -1$

d) $\lim_{x\to -5^-} f(x) \neq \lim_{x\to -5^+} f(x)$. Therefore $\lim_{x\to -5} f(x) = \text{DNE}$

e) $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$. Therefore $\lim_{x\to 5} f(x) = 1$

6.

a) $\lim_{x\to -2^-} f(x) = 3$

b) $\lim_{x\to -2^+} f(x) = 4$

c) $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$. Therefore $\lim_{x\to -2} f(x) = \text{DNE}$

d) $\lim_{x\to -5^-} f(x) \neq \lim_{x\to -5^+} f(x)$. Therefore $\lim_{x\to -5} f(x) = \text{DNE}$

e) $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$. Therefore $\lim_{x\to 5} f(x) = 2$

7.

a) $\lim_{x\to -2^-} f(x) = 0$

b) $\lim_{x\to -2^+} f(x) = 0$

c) $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$. Therefore $\lim_{x\to -2} f(x) = 0$

d) $\lim_{x\to -4^-} f(x) \neq \lim_{x\to -4^+} f(x)$. Therefore $\lim_{x\to -4} f(x) = \text{DNE}$

e) $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$. Therefore $\lim_{x\to 5} f(x) = 3$

8.

a) $\lim_{x\to -2^-} f(x) = 0$

b) $\lim_{x\to -2^+} f(x) = 1$

c) $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$. Therefore $\lim_{x\to -2} f(x) = \text{DNE}$

d) $\lim_{x\to -3^-} f(x) \neq \lim_{x\to -3^+} f(x)$. Therefore $\lim_{x\to -3} f(x) = \text{DNE}$

e) $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$. Therefore $\lim_{x\to 5} f(x) = -1$

9.

a) $\lim_{x\to 2^-} f(x) = -1$

b) $\lim_{x\to 2^+} f(x) = -1$

c) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$. Therefore $\lim_{x\to 2} f(x) = -1$

d) $\lim_{x\to -4^-} f(x) \neq \lim_{x\to -4^+} f(x)$. Therefore $\lim_{x\to -4} f(x) = \text{DNE}$

e) $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$. Therefore $\lim_{x\to 4} f(x) = -1$

10.

a) $\lim_{x\to -2^-} f(x) = 3$

b) $\lim_{x\to -2^+} f(x) = 3$

c) $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$. Therefore $\lim_{x\to -2} f(x) = 3$

d) $\lim_{x\to -3^-} f(x) \neq \lim_{x\to -3^+} f(x)$. Therefore $\lim_{x\to -3} f(x) = \text{DNE}$

e) $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$. Therefore $\lim_{x\to 3} f(x) = -3$