Lesson 4: Finding Limits Analytically

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3 different cases:

① $f(c)$ returns a #

i.e. $f(x)$ is continuous @ $x=c$

i.e. $\lim_{x \to c} f(x) = f(c)$

Ex 1: $\lim_{x \to 4} (2x-3) = 2(4)-3$

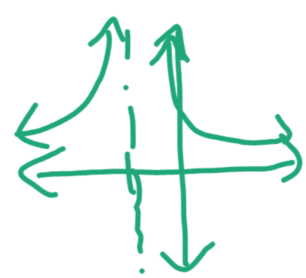
$= 8-3 = \boxed{5}$

② $f(c)$ returns $\frac{\text{nonzero \#}}{0}$.

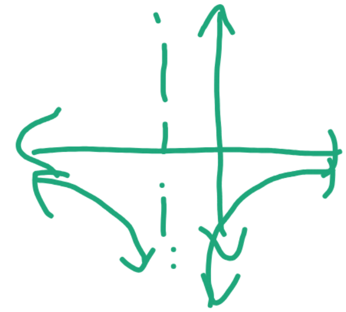
i.e. Vertical Asymtote @ $x=c$

i.e. $\lim_{x \to c} f(x) = \pm\infty$ or DNE

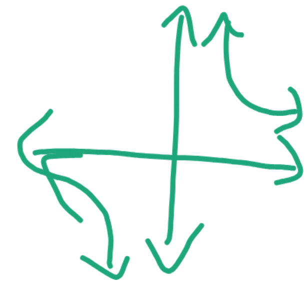
Ex: $\lim_{x \to -1} \frac{1}{(x+1)^2} = \infty$



Ex: $\lim_{x \to -1} \frac{-1}{(x+1)^2} = -\infty$



Ex: $\lim_{x \to 0} \frac{1}{x}$ $\boxed{\text{DNE}}$



③ $f(c)$ returns $0/0$

i.e. $f(x)$ has a hole → If factor cancels out

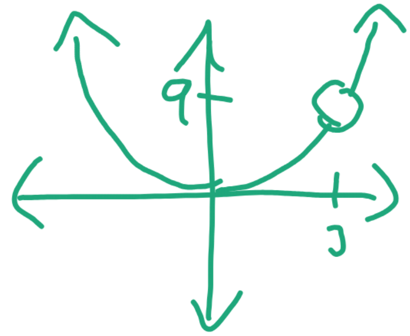
or VA @ $x=c$. → Doesn't

What could happen is limit not exist.

Idea: Manipulate $f(x)$ to be Case 1 or 2.

Ex: $\lim_{x \to 3} \frac{x^3 - 3x^2}{x-3} = \frac{3^3 - 3 \cdot 3^2}{3-3} = \frac{3^3 - 3^3}{3-3} = \frac{0}{0}$

$= \lim_{x \to 3} \frac{x^2 \cancel{(x-3)}}{\cancel{x-3}} = \lim_{x \to 3} x^2 = 9$



Ex: $\lim_{x\to 2} \frac{x^2-2x}{(x-2)^2} = \frac{2^2-2\cdot 2}{(2-2)^2} = \frac{0}{0}$

$= \lim_{x\to 2} \frac{x\cancel{(x-2)}}{(x-2)^{\cancel{2}}} = \lim_{x\to 2} \frac{x}{x-2} = \frac{2}{0}$

$\frac{x-2+2}{x-2} = \frac{x-2}{x-2} + \frac{2}{x-2} = 1 + \underbrace{\frac{2}{x-2}}_{\text{DNE}}$

$\boxed{\text{Final Answer: } \lim_{x\to 2} \frac{x^2-2x}{(x-2)^2} \text{ DNE}}$

Ex: $f(x) = \begin{cases} 1/x & \text{if } x \geq -2 \\ -x & \text{if } x < -2 \end{cases}$

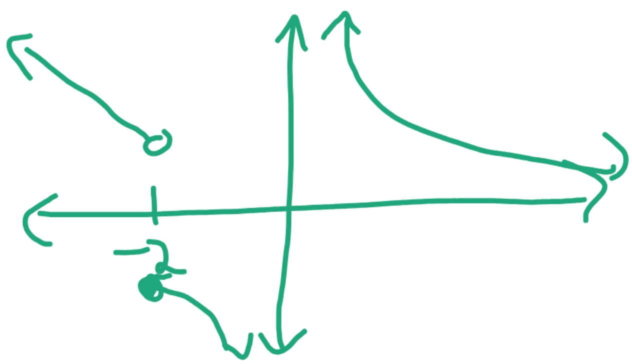
(a) $\lim_{x\to -2} f(x)$ DNE

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2} (-x) = -(-2) = 2$

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2} \left(\frac{1}{x}\right) = -\frac{1}{2}$ $\neq$

$$f(x) = \begin{cases} 1/x & \text{if } x \geq -2 \\ -x & \text{if } x < -2 \end{cases}$$

(b) $\lim\limits_{x \to 0} f(x) = \boxed{\lim\limits_{x \to 0} \frac{1}{x} \quad \text{DNE}}$



Ex: $f(x) = \begin{cases} \sin x & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$

$\lim\limits_{x \to 0} f(x) = \boxed{0}$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0} x^2 = 0^2 = 0$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0} \sin x = \sin 0 = 0$

Prop. of Limits

Let c, k, L, and K be real numbers and n a positive integer. If

$$\lim_{x \to c} f(x) = L \qquad \lim_{x \to c} g(x) = K$$

- $\lim_{x \to c} [k f(x)] = kL$

  $\hookrightarrow k\left[\lim_{x \to c} f(x)\right] = kL$

- $\lim_{x \to c} [f(x) \pm g(x)]$

  $= \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = L \pm K$

- $\lim_{x \to c} [f(x) g(x)]$

  $= \left(\lim_{x \to c} f(x)\right)\left(\lim_{x \to c} g(x)\right) = L \cdot K$

- $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$ assuming $K \neq 0$

- $\lim_{x \to c} [f(x)]^n = L^n$