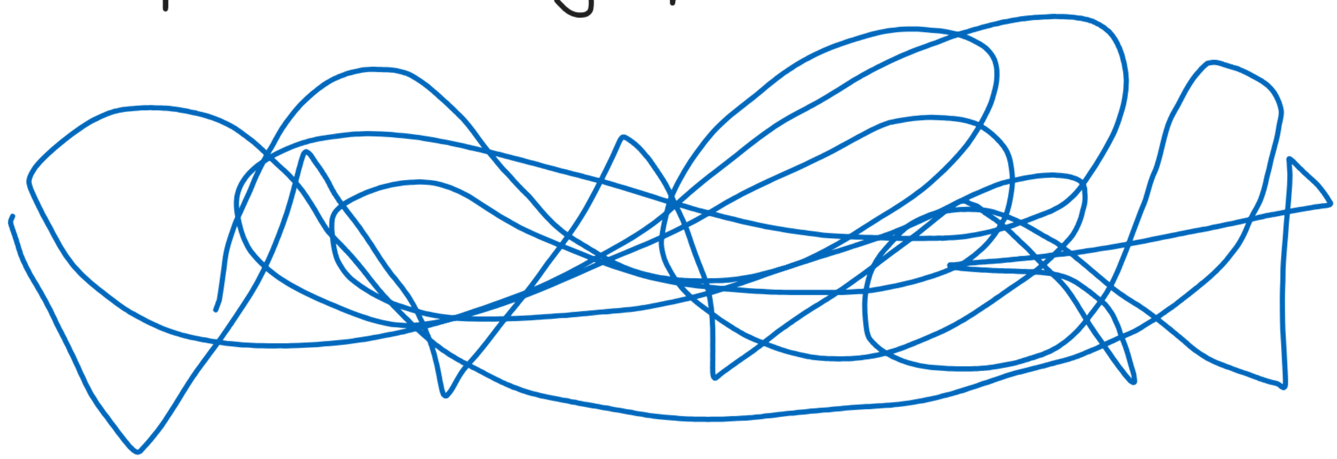
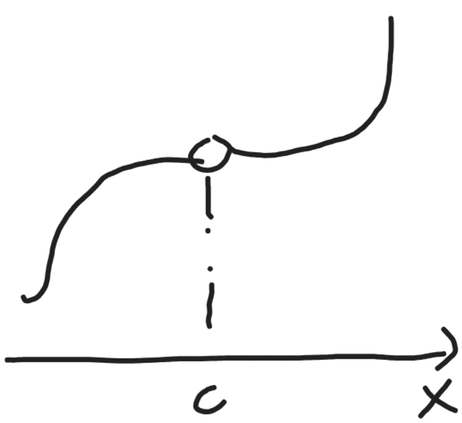
Lesson 5: Contunity

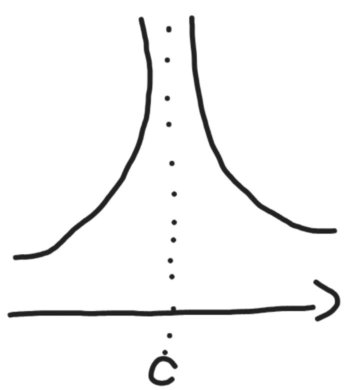
Lesson 5: Continuity

a function is continuous if there is no disruption in the graph.

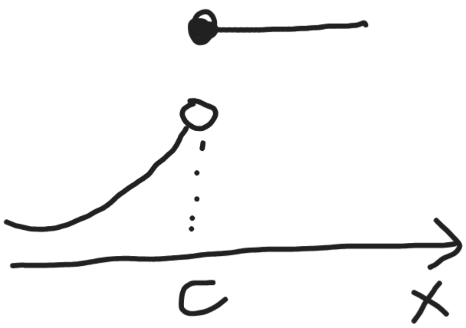


The following 4 graphs show $f(x)$ is discontinuous @ $x=c$.



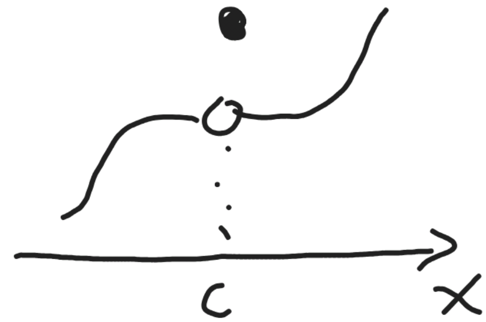


These 2 graph have $f(c)$ undefind



$f(c)$ is defined

$\lim\limits_{x \to c} f(x)$ DNE



$f(c)$ is defined

$\lim\limits_{x \to c} f(x)$ exists

BUT $\neq f(c)$

We can see a function $f(x)$ is continuous @ $x=c$ if the following is true:

① $f(c)$ defined (has a value)

② $\lim\limits_{x \to c} f(x)$ exists

③ $\lim\limits_{x \to c} f(x) = f(c)$

If any of the 3 conditions aren't met, then we say $f(x)$ is discontinuous @ $x=c$.

Ex 1: Discuss continuity of $f(x) = \frac{2x}{x^2 - x}$

Set denominator to 0.

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0,\ x = 1$

$f(x) = \frac{\cancel{2x}}{\cancel{x}(x-1)}$

$x = 0 \Rightarrow$ hole

(You want cancel-ation)

$x = 1 \Rightarrow$ VA

HW 5.4:

$f(x) = \frac{x^2 + 2x - 3}{x^2 + 5x - 6}$

-3

$-1 \wedge 3$

$x^2 + 2x - 3$

$= x^2 + 3x - x - 3$

$= x(x+3) - (x+3)$

$= (x-1)(x+3)$

$x^2 + 5x - 6$

-6

$6 \wedge -1$

$= x^2 + 6x - x - 6$

$= x(x+6) - (x+6)$

$= (x-1)(x+6)$

$$f(x) = \frac{x^2+2x-3}{x^2+5x-6} = \frac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}(x+6)}$$

If any thing cancels I have a hole.

$x-1=0 \Rightarrow x=1$ (hole)

what remains in denominator.

$x+6=0 \Rightarrow x=-6$ (VA)

Ex 2: $f(x) = \begin{cases} e^{-x} & , \text{if } x \le 0 \\ \sqrt{x}-1 & \text{if } x > 0 \end{cases}$

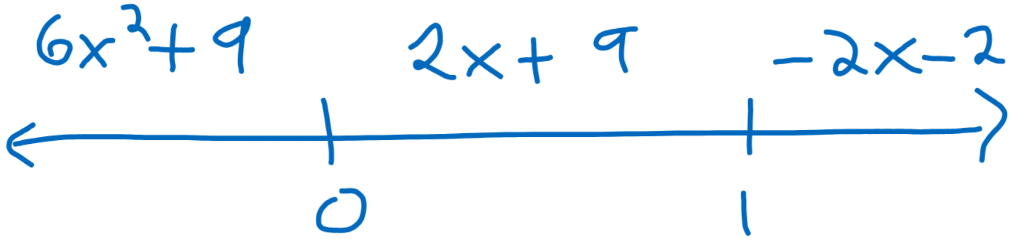
Check: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} e^{-x} = e^{-0} = 1$ $\neq$ Discontinuous $\Rightarrow$ Jump

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} (\sqrt{x}-1) = \sqrt{0}-1 = -1$

HW 5.8:

$f(x) = \begin{cases} 6x^2+9 & x \le 0 \\ 2x+9 & 0 < x < 1 \\ -2x-2 & x \ge 1 \end{cases}$



$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} (6x^2+9) = 9$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} (2x+9) = 9$$

Not a discontinuity

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (2x+9) = 2+9 = 11$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (-2x-2) = -2-2 = -4$$

$\neq$ Jump