Lesson 7: Basic Rules of Differentiation; Derivatives of Sine and Cosine Functions; Derivatives of the Natural Exponential Function

Lesson 7.

- **Basic Rules of Differentiation**

① Constant Rule: For any constant c,

$$\frac{d}{dx}(c) = 0$$

Intuitively makes sense b/c graph $f(x) = c$ is a horizontal line.

Proof: $f(x) = c$

$$\begin{aligned} f(x+h) &= c \\ -f(x) &= -c \\ \hline f(x+h) - f(x) &= 0 \end{aligned}$$

$$\underbrace{\frac{f(x+h) - f(x)}{h}}_{} = \frac{0}{h} = 0$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 0 = \boxed{0}$$

Power Rule: For any real number, n,

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

Ex. $f(x) = x^2$
$f'(x) = 2x^{2-1} = 2x$

$f(x) = x^{-4}$
$f'(x) = -4x^{-4-1} = -4x^{-5}$

Constant Multiple Rule:

$$\frac{d}{dx}[c \cdot f(x)] = c \frac{d}{dx}[f(x)]$$

Sum/Difference Rule:

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

Ex 3: $f(x) = x^5 + 5x^2$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}[f(x)] = \frac{d}{dx}[x^5 + 5x^2]$$

$$= \frac{d}{dx}[x^5] + \frac{d}{dx}[5x^2]$$

$$= \frac{d}{dx}[x^5] + 5\frac{d}{dx}[x^2] = 5x^4 + 5 \cdot 2x$$

$$= 5x^4 + 10x$$

Derivatives share the same properties as limit w/ the exception of product and quotient.

Ex 4: $f(x) = \frac{3}{x^4} - 2x^2 + 6x - 7$.

$\boxed{x^{-m} = \frac{1}{x^m}}$

$f(x) = 3x^{-4} - 2x^2 + 6x - 7$

$$f'(x) = \frac{d}{dx}\left[3x^{-4} - 2x^2 + 6x - 7\right]$$

$$= 3\frac{d}{dx}\left[x^{-4}\right] - 2\frac{d}{dx}\left[x^2\right] + 6\frac{d}{dx}[x] - \cancel{\frac{d}{dx}[7]}^{\,0}$$

$$= 3(-4)x^{-4-1} - 2\cdot 2x^{2-1} + 6 - 0$$

$$= -12x^{-5} - 4x + 6$$

$$= -\frac{12}{x^5} - 4x + 6$$

- Derivative of sinx and cosx

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

Important Limits

1. $$\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$
2. $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

HW 7.10: $y = 3\sin x - 4\cos x$

$$y' = \frac{d}{dx}[3\sin x - 4\cos x]$$

$$= 3\underbrace{\frac{d}{dx}[\sin x]}_{\cos x} - 4\underbrace{\frac{d}{dx}[\cos x]}_{-\sin x}$$

$$= 3\cos x + 4\sin x$$

- Derivatives of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Important Limit: $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

HW 7.14: Find the x-value @ which the derivative of $y = 10e^x$ is 1.

$y' = 1 \rightarrow$ solve for x.

$y' = \frac{d}{dx}(10e^x) = 10\frac{d}{dx}(e^x) = 10e^x = 1$

$e^x = 1/10$

$\ln(e^x) = \ln(1/10)$

$x = \ln(1/10)$

$\ln(1/10) = \cancel{\ln(1)}^{\,0} - \ln(10)$

$= -\ln(10)$