Lesson 8: Instantaneous Rates of Change

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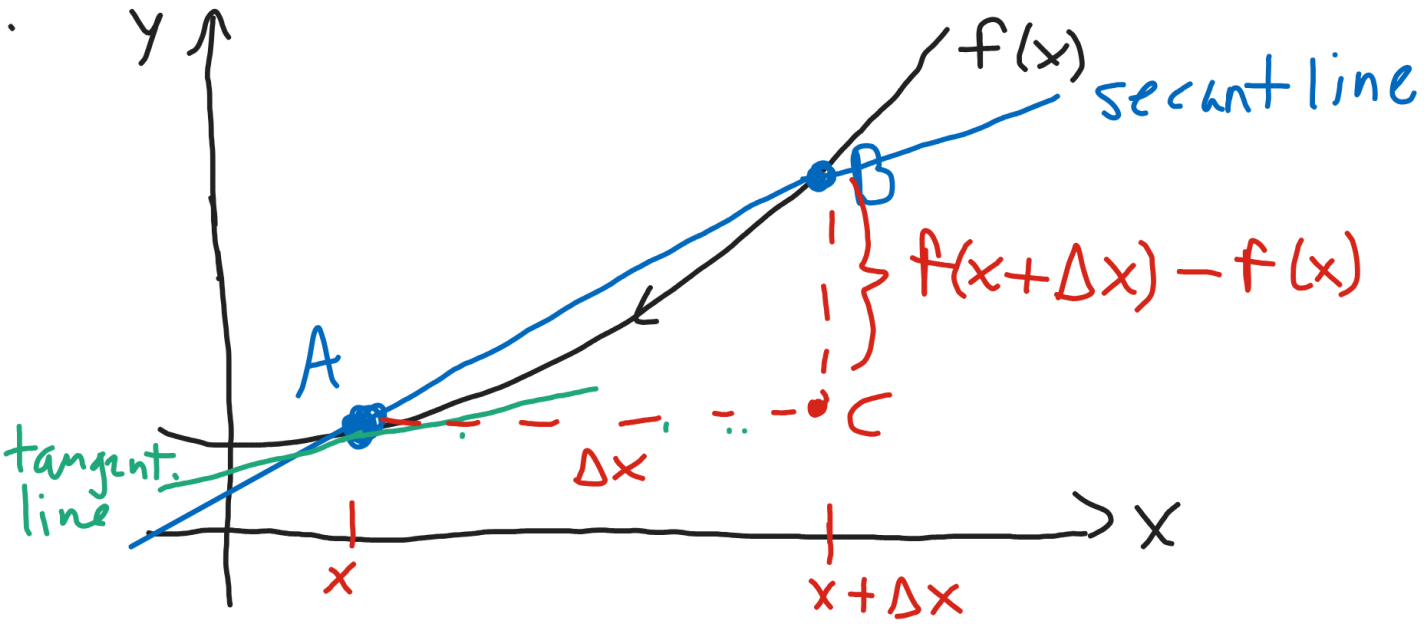


Slope of secant line: $\frac{f(x+\Delta x)-f(x)}{\Delta x}$

This quantity is also known as the average rate of change.

Average rate of change → a quantity is called instantaneous rate of change

↳ Derivative

Ex1: The initial pop of a culture of bacteria is 1000. The pop after t hours, $P(t)$, is given by

$$P(t) = 2t^2 + 8t + 1000$$

1) Find the # of bacteria present after 5 hrs.

$P(5) = 2(5)^2 + 8(5) + 100 = 1090$

2) Find the rate of change of the pop after 5 hrs.

$P'(t) = 4t + 8$

$P'(5) = 4(5) + 8 = 28$

3) Interpret the answer in Pt 2.

after 5 hrs, the pop grew @ a rate of 28 bacteria per hour.

Position & Velocity Function

Position [$s(t)$] tells us how far away an object is

Velocity [$v(t)$] tells us speed of an object w/ respect to direction

Derivative

$$v(t) = s'(t)$$

Ex 3. An object is shot upward from the surface of Earth. The position function is

$$s(t) = -4.9t^2 + 98t$$

1) $v(t) = ?$

$v(t) = s'(t) = -4.9(2)t + 98 = -9.8t + 98$

2) $v(3) = ?$

$v(3) = -9.8(3) + 98 = 68.6$

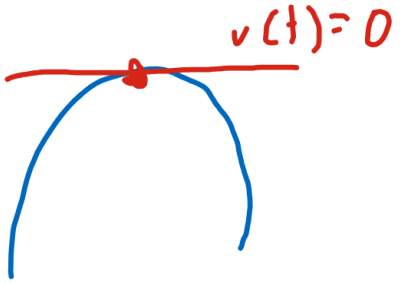
3) What is the velocity of the object when it hits the ground?

Solve $s(t) = 0$ for t. Plug t into $v(t)$.

$$0 = s(t) = -4.9t^2 + 98t$$
$$= -4.9t(t - -20)$$
$$-4.9t = 0 \quad | \quad t - 20 = 0$$
$$t = 0 \quad | \quad t = 20$$

$v(20) = -9.8(20) + 98 = \boxed{-98}$

(4) When is the object at its highest pt?



$-9.8t + 98 = 0$

$\frac{98}{9.8} = \frac{9.8t}{9.8}$

$t = 10$

Ex 4: $C = 2\pi r$. What is the range of change of C w/ respect to r?

$dC/dr = ?$

$\frac{d}{dr}[C] = \frac{d}{dr}[2\pi r]$

$\frac{dC}{dr} = 2\pi \frac{dr}{dr} = \boxed{2\pi}$

Ex 5: $p = 3q - 5$

(a) What is the rate of change of p w/ respect to q? $[dp/dq = ?]$

$$\frac{d}{dq}[p] = \frac{d}{dq}[3q - 5]$$

$$\frac{dp}{dq} = 3\frac{dq}{dq} - \frac{d}{dq}[5] \quad (\frac{d}{dq}[5] \to 0)$$

$$\frac{dp}{dq} = 3$$

(b) What is the rate of change of q w/ respect to p? $[dq/dp = ?]$

$$\frac{d}{dp}[p] = \frac{d}{dp}[3q - 5]$$

$$\frac{dp}{dp} = 3\frac{dq}{dp} - \frac{d}{dp}[5] \quad (\frac{d}{dp}[5] \to 0)$$

$$1 = 3\frac{dq}{dp} \iff \frac{1}{3} = \frac{dq}{dp}$$

(c) What is the rate of change of [illegible] w/ [illegible]