Lesson 9: Product Rule

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$h(x) = f(x)\,g(x)$

$$\frac{d}{dx}[h(x)] = \frac{d}{dx}[f(x)g(x)]$$

$$= \frac{d}{dx}[f(x)]\,g(x) + f(x)\frac{d}{dx}[g(x)]$$

$$= f'(x)\,g(x) + f(x)\,g'(x)$$

Ex 1: $h(x) = x^2 \sin x$. Compute $h'(\pi/6)$.

$f(x) = x^2$ $\quad$ $g(x) = \sin x$

$f'(x) = 2x$ $\quad$ $g'(x) = \cos x$

$h'(x) = f'g + g'f$

$= 2x\sin x + \cos x \cdot x^2$

$$h'\left(\frac{\pi}{6}\right) = 2\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right)\cdot\left(\frac{\pi}{6}\right)^2$$

$$= \frac{\pi}{3}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\pi^2}{36} = \frac{\pi}{6} + \frac{\sqrt{3}\,\pi^2}{72}$$

Ex 2: $h(x) = 2x^3 e^x$. Find $h'(x)$.

$f(x) = 2x^3 \quad g(x) = e^x$

$f'(x) = 6x^2 \quad g'(x) = e^x$

$h'(x) = f'(x)g(x) + g'(x)f(x)$

$= 6x^2 e^x + 2x^3 e^x$

Ex 3: $h(x) = \sqrt{x}(2x^2+4)$

Method 1: (Product Rule)

$f(x) = \sqrt{x} = x^{1/2} \quad g(x) = 2x^2+4$

$f'(x) = \frac{1}{2}x^{-1/2} \quad g'(x) = 4x$

$h'(x) = f'(x)g(x) + g'(x)f(x)$

$= \frac{1}{2}x^{-1/2}(2x^2+4) + 4x \cdot x^{1/2}$

$= x^{2-1/2} + 2x^{-1/2} + 4x^{1+1/2}$

$= x^{3/2} + 2x^{-1/2} + 4x^{3/2}$

$= \boxed{5x^{3/2} + 2x^{-1/2}}$

Method 2: $h(x) = \sqrt{x}(2x^2+4)$

$= x^{1/2}(2x^2+4)$

$= 2x^{5/2} + 4x^{1/2}$

$$h'(x) = 2\cdot\frac{5}{2}x^{3/2} + 4\left(\frac{1}{2}\right)x^{-1/2} = \boxed{5x^{3/2} + 2x^{-1/2}}$$

HW 1.2: $h(x) = (x^2+5x)(-3x^5+1)$

	x^2	$5x$
$-3x^5$	$-3x^7$	$-15x^6$
6	$6x^2$	$30x$

$$= -3x^7 - 15x^6 + 6x^2 + 30x$$

$$h'(x) = -21x^6 - 15\cdot 6x^5 + 12x + 30$$

$$= -21x^6 - 90x^5 + 12x + 30$$

$$h(x) = (2x^2+3x+2)(2x+1)$$

	$2x^2$	$3x$	2
$2x$	$4x^3$	$6x^2$	$4x$
1	$2x^2$	$3x$	2

$$h(x) = 4x^3 + 8x^2 + 7x + 2$$

HW 9.9: Find x values @ which $y = 4x^6 e^x$ has a horizontal tangent.

↳ $y'(x) = 0$ solve for x.

$f(x) = 4x^6$ $\quad g(x) = e^x$

$f'(x) = 24x^5$ $\quad g'(x) = e^x$

$y' = f'(x)g(x) + g'(x)f(x)$

$= 24x^5 e^x + e^x 4x^6 = 0$

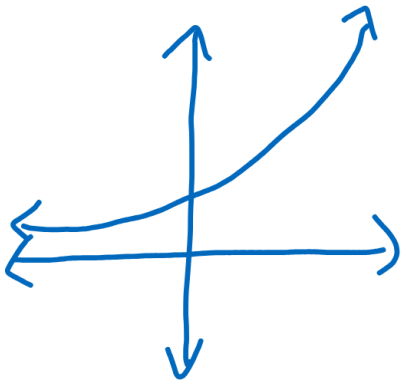
$4x^5 e^x (6 + x) = 0$

$4x^5 = 0$ | $e^x = 0$ | $6 + x = 0$

$x = 0$ | ↓ | $x = -6$

Never happens

Note $e^x \neq 0$



Final Answers

$x = 0, \; x = -6$