10/12/22 | MA 16020 : Exam 2 Review Problems

(1) Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \frac{\sin x}{1-\cos x}\,dx$$

Recall an integral is improper if the bound(s) have an infinity ($\pm\infty$) or if the integrand is undefined in the interval of the bounds.

So no '∞', implies we need to check the second case. i.e. Is $\frac{\sin x}{1-\cos x}$ defined on the interval $[0,\frac{\pi}{2}]$?

A fraction is undefined when the denominator $=0$.

$$1-\cos x = 0$$
$$1 = \cos x$$
$$x = 0, 2\pi, \ldots$$

Since 0 is in $[0, \pi/2]$, we have an improper integral.

Answer: Improper, since the integrand is undefined at 0.

10/12/22 | MA16020: Exam 2 Review Problems

② Evaluate the following integral:

$$\int_0^\infty e^{-3x}\,dx$$

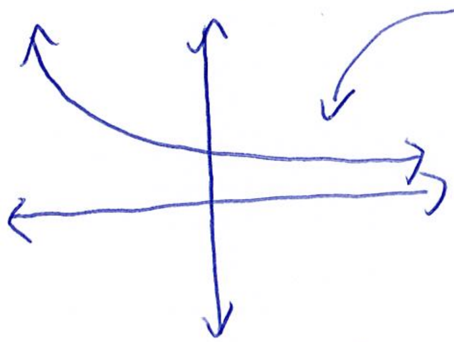
$$\int_0^\infty e^{-3x}\,dx = \lim_{N\to\infty}\int_0^N e^{-3x}\,dx$$

$$= \lim_{N\to\infty}\left(-\frac{1}{3}e^{-3x}\Big]_0^N\right)$$

$$= \lim_{N\to\infty}\left(-\frac{1}{3}e^{-3N} + \frac{1}{3}e^{-3(0)}\right)$$

$$= \lim_{N\to\infty}\left(-\frac{1}{3}e^{-3N} + \frac{1}{3}\right)$$

Recall the graph of e^{-3x}



So as $x\to\infty$, $e^{-3x}\to 0$
i.e. $N\to\infty$, $e^{-3N}\to 0$

Hence $\int_0^\infty e^{-3x}\,dx = -\frac{1}{3}\cdot 0 + \frac{1}{3} = \frac{1}{3}$

Answer: $1/3$