MA1602O Exam 2 Study Guide: Calculus

An improper integral is when
(1) we have $\pm\infty$ in the bounds, or
(2) we have a discontinuity within the bounds.

Check if the integrand is undefined and check if that value is in the interval.

When computing them, rewrite with a limit

example: $\int_0^{\infty} e^{-x}\,dx = \lim_{N\to\infty} \int_0^{N} e^{-x}\,dx$

Area Between Two Curves

The area between two curves can be described two ways:

$A = \int_a^b (\text{Top} - \text{Bottom})\,dx$ → You want y = something x for Top and Bottom

or $A = \int_c^d (\text{Right} - \text{Left})\,dy$ → You want x = something y for Right and Left

Volume of Solids of Revolution

Read the problem to see if a particular metho is asked for. Plus try to draw the regions.

When the region "hugs" the line of rotation ⇒ Disk

- x-axis ⇒ dx-problem ⇒ $V=\int_a^b \pi(f(x))^2\,dx$
- y-axis ⇒ dy-problem ⇒ $V=\int_c^d \pi(g(y))^2\,dy$
- the line $y=\#$ ⇒ dx-problem ⇒ $V=\int_a^b \pi(f(x)-\#)^2\,dx$
- the line $x=\#$ ⇒ dy-problem ⇒ $V=\int_c^d \pi(g(y)-\#)^2\,dy$

When there is a "gap" between the region and the line of rotation ⇒ Washer

- x-axis ⇒ dx-problem ⇒ $V=\int_a^b \pi(R^2-r^2)\,dx$
- y-axis ⇒ dy-problem ⇒ $V=\int_c^d \pi(R^2-r^2)\,dy$
- the line $y=\#$ ⇒ dx-problem ⇒ $V=\int_a^b \pi\left[(R-\#)^2-(r-\#)^2\right]dx$
- the line $x=\#$ ⇒ dy-problem ⇒ $V=\int_a^b \pi\left[(R-\#)^2-(r-\#)^2\right]dy$

where R is the farthest from the line of rotation and r is the closest to the line of rotation.

But if you find solving for x or y, in either method, is hard ⇒ Shell

- x-axis ⇒ dy-problem ⇒ $V = \int_c^d 2\pi y \cdot g(y)\,dy$
- y-axis ⇒ dx-problem ⇒ $V = \int_a^b 2\pi x \cdot f(x)\,dx$

Growth & Decay Differential Equations

- Proportional to population ⇒ $y' = ky \Rightarrow y = Ce^{kt}$
- Half-life Problems ⇒ $y = Ce^{kt}$ with

$$k = \frac{\ln(1/2)}{\text{half-life}} = \frac{-\ln(2)}{\text{half-life}}$$

Separation of Variables

Solve differential equations of the type

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

The idea is to try to get terms w/ y on one-side and x-terms on the other. Then integrate and solve for y.