

## Reminders

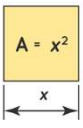
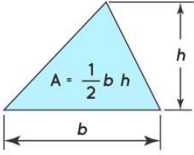
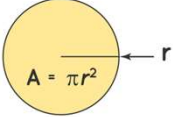
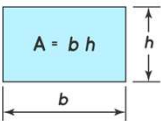
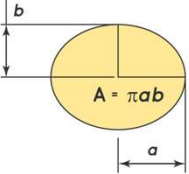
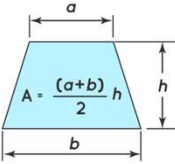
- TODAY QUIZ 4 on Area Between Two Curves

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## MA 16020: Lesson 14 Volume By Revolution Disk Method

By: Alexandra Cuadra

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<p><b>Square</b></p>  <p><math>A = x^2</math></p>	<p><b>Triangle</b></p>  <p><math>A = \frac{1}{2} b h</math></p>	<p><b>Circle</b></p>  <p><math>A = \pi r^2</math></p>
<p><b>Rectangle</b></p>  <p><math>A = b h</math></p>	<p><b>Eclipse</b></p>  <p><math>A = \pi a b</math></p>	<p><b>Trapezoid</b></p>  <p><math>A = \frac{(a+b)}{2} h</math></p>

**In Geometry,**

When we first talked about the concept of area, we did this by going over all the formulas for the area of different polygons.

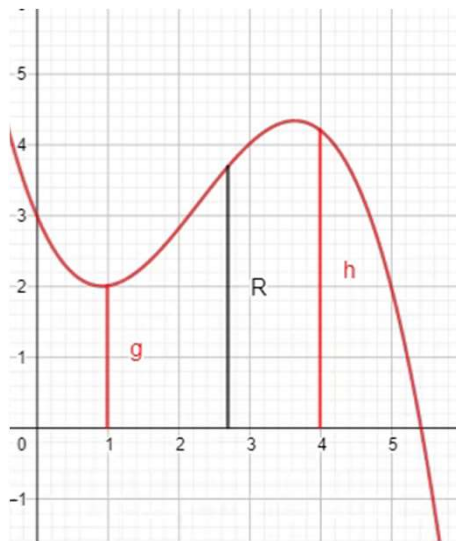
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## In Calculus I,

We learned about integration as a new technique for calculating area under a curve.

$$\text{i.e. } \int_a^b f(x) dx = F(b) - F(a)$$

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<https://www.geogebra.org/m/tgceabb2#material/tnnhu7gz>

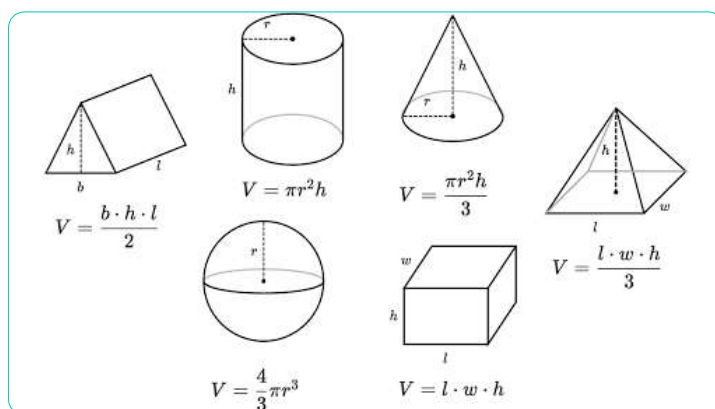
## Last Class,

- We recapped how to take some region
  - Between a curve and an axis, or
  - 2 curves
 And find it's area by integration.
- Essentially, finding a length and sending it across the region.

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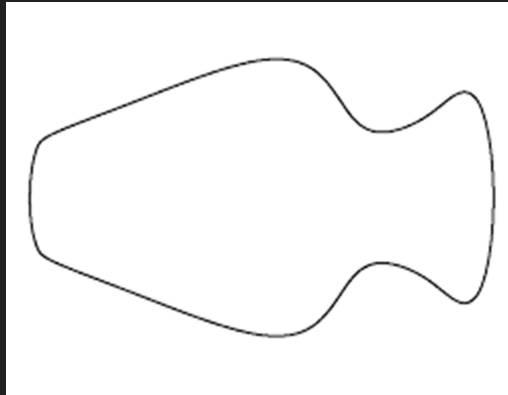
## In Geometry,

- We also learned about 3-D figures, like cubes and prisms.
- We described the volume of these objects by the amount of 3-D space that they contained.
- We calculated the volume with formulas like the ones on the right.

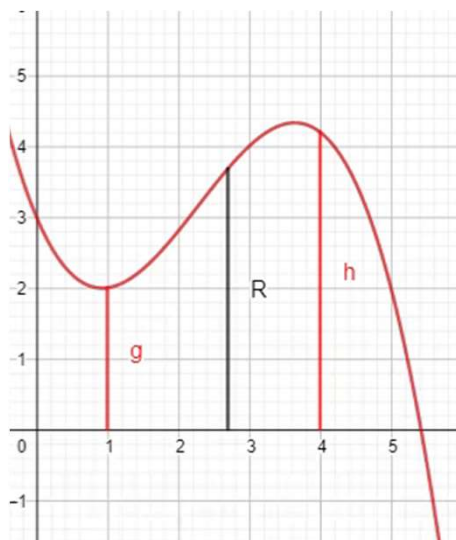


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But once curves, like the one below, get involved all these formulas are **USELESS**.



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<https://www.geogebra.org/m/tgceabb2#material/tnnhu7gz>

○ In the same way, we run a line segment across a 2-D region to calculate its area, we can run a plane region, or a cross section, across a 3-D region to calculate its volume.

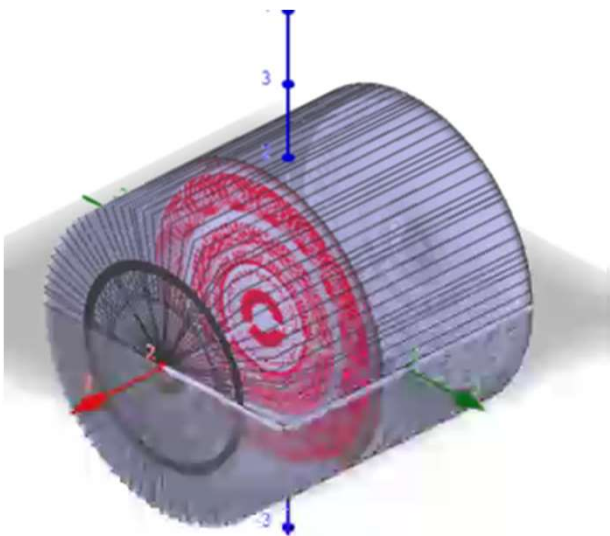
○ i.e. Running a 2-D plane across a 3-D volume.

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## So we have integration again; just with an extra dimension.

- Instead of adding up tiny rectangles under a curve, we are adding up infinitely thin cross sections, which we can call
  - Disks (Lesson 14), or
  - Washers (Lesson 15), or
  - Shells (Lesson 17)
  
- Since each of these cross sections are 2-D, taking the integral of an area function will give us volume.

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<https://www.geogebra.org/m/tgceabb2#material/qcwutumt>

### Let's look at a cylinder.

Remember a cylinder is made up of many circles like the **red circle**.

So, we can think of our integral to be sum of all these circles.

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## Volume of that Cylinder

- Geometry Way:

- The formula for a Cylinder is

$$V = \pi r^2 h$$

- Since our Cylinder has radius 2 and height 4,

$$V = \pi 2^2 4 = 16 \pi$$

- Calculus Way:

$$V = \int_{-2}^2 2^2 \pi dx = 16 \pi$$

where  $\int_{-2}^2$  refers to the height, and

$2^2 \pi$  refers to the area of a circle.

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## Purpose of all of this...

- So in the case of a cylinder, this might be overkill.
- But this is the way we want to think of these questions.
- Essentially find the cross section by graphing the lines given and apply the appropriate formula ( found on the next slide. )

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## Disk Method Formula(s)

For rotation around  $x$ -axis:

- If the volume of the solid is obtained by rotating  $f(x)$  about the  $x$ -axis on the interval  $a \leq x \leq b$  is given by

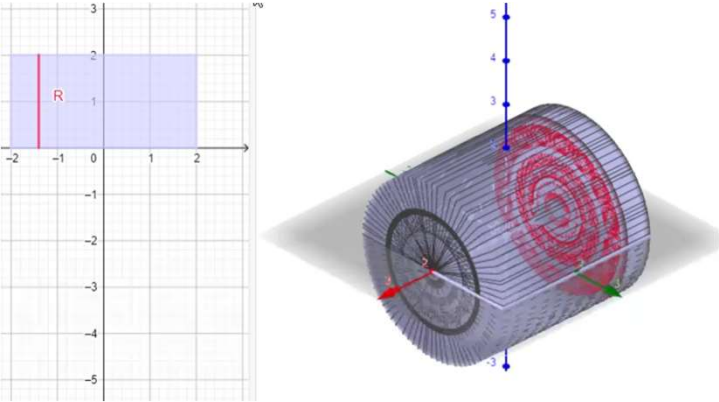
$$V = \pi \int_a^b [f(x)]^2 dx$$

For rotation around  $y$ -axis:

- If the volume of the solid is obtained by rotating  $g(y)$  about the  $y$ -axis on the interval  $c \leq y \leq d$  is given by

$$V = \pi \int_c^d [g(y)]^2 dy$$

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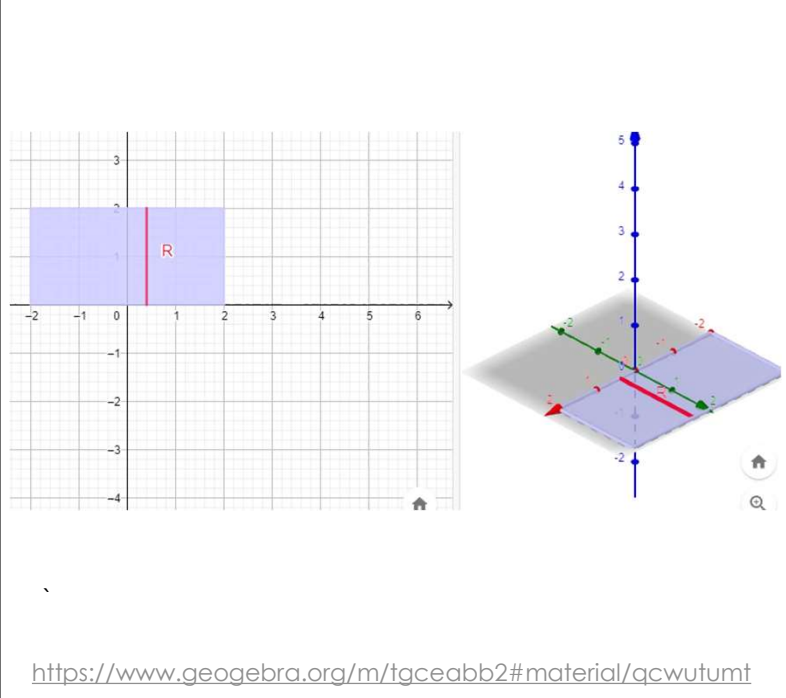
### Why $\pi$ in the formula?

Note the  $\pi$  in both formulas comes from the fact we are playing with Disks.

So you can see the graph on the left shows the radius and the right shows the Disks.

<https://www.geogebra.org/m/tgceabb2#material/qcwutumt>

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**But wait...**

How is the left graph becoming the cylinder?

Answer: If you rotate the left graph, you get the cylinder.

<https://www.geogebra.org/m/tgceabb2#material/qcwutumt>

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# Examples

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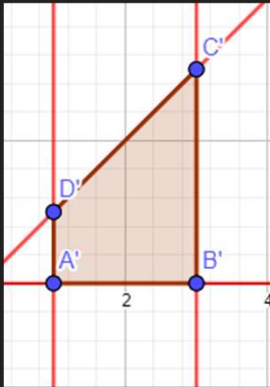


Example 1: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = x, \quad y = 0, \quad x = 1, \quad x = 3$$

About the x-axis.

First draw the region.



<https://www.geogebra.org/m/tgceabb2#material/w8mk9dgp>

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Example 1: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = x, \quad y = 0, \quad x = 1, \quad x = 3$$

About the x-axis.

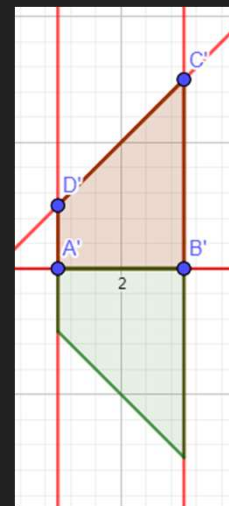
Example 1: Find the volume of the solid that results by revolving the region enclosed by the

curves

$$y = x, \quad y = 0, \quad x = 1, \quad x = 3$$

About the x-axis.

Rotation about x-axis



<https://www.geogebra.org/m/tgceabb2#material/w8mk9dgp>

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Example 1: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = x, \quad y = 0, \quad x = 1, \quad x = 3$$

About the x-axis.

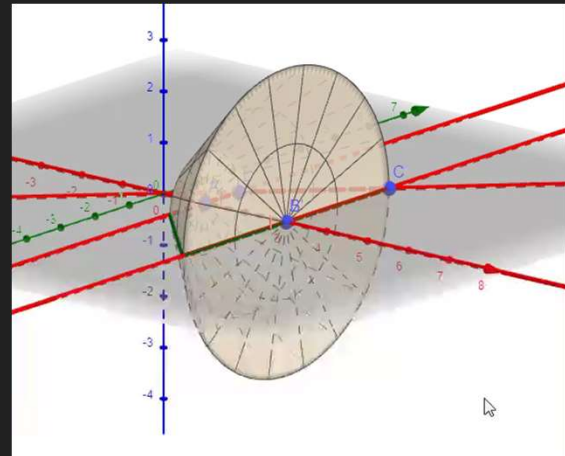
Example 1: Find the volume of the solid that results by revolving the region enclosed by the

curves

$$y = x, \quad y = 0, \quad x = 1, \quad x = 3$$

About the x-axis.

Furthermore, 3-D



<https://www.geogebra.org/m/tgceabb2#material/w8mk9dgp>

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Example 1: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = x, \quad y = 0, \quad x = 1, \quad x = 3$$

About the x-axis.

$$V = \pi \int_1^3 (x)^2 dx$$

$$= \pi \left[ \frac{x^3}{3} \right]_1^3$$

$$= \pi \left( \frac{3^3}{3} - \frac{1}{3} \right) = \frac{26}{3} \pi$$

Example 1: Find the volume of the solid that results by revolving the region enclosed by the

curves

$$y = x, \quad y = 0, \quad x = 1, \quad x = 3$$

About the x-axis.

<https://www.geogebra.org/m/tgceabb2#material/w8mk9dgp>

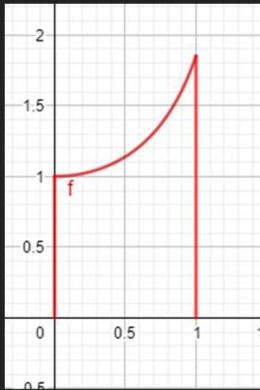
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Example 2: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sec(x), \quad y = 0, \quad x = 0, \quad x = 1$$

About the x-axis.

First draw the region.



<https://www.geogebra.org/m/tgceabb2#material/vte3zdjx>

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Example 2: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sec(x), \quad y = 0, \quad x = 0, \quad x = 1$$

About the x-axis.

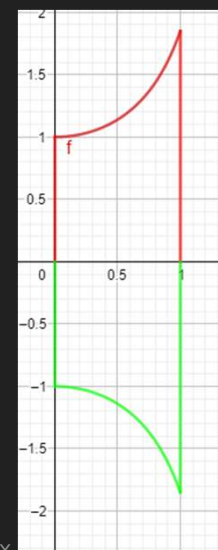
Example 2: Find the volume of the solid that results by revolving the region enclosed by the

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$$y = \sec(x), \quad y = 0, \quad x = 0, \quad x = 1$$

About the x-axis.

Rotation about x-axis



<https://www.geogebra.org/m/tgceabb2#material/vte3zdjx>

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Example 2: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sec(x), \quad y = 0, \quad x = 0, \quad x = 1$$

About the x-axis.

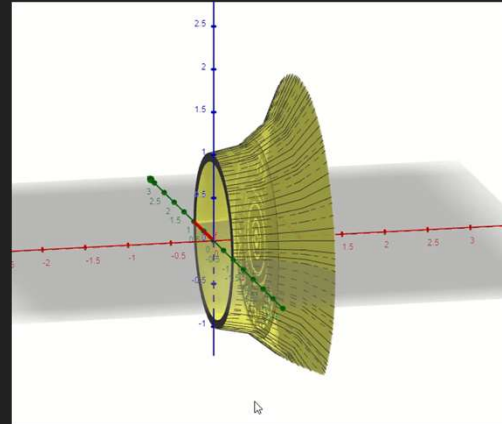
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$$y = \sec(x), \quad y = 0, \quad x = 0, \quad x = 1$$

About the x-axis.

Furthermore, 3-D



<https://www.geogebra.org/m/tgceabb2#material/vte3zdjx>

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Example 2: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sec(x), \quad y = 0, \quad x = 0, \quad x = 1$$

About the x-axis.

$$V = \pi \int_0^1 (\sec x)^2 dx$$

$$= \pi [\tan x]_0^1$$

$$= \pi (\tan(1) - \tan(0))$$

$$= \pi \tan(1)$$

Example 2: Find the volume of the solid that results by revolving the region enclosed by the

curves

$$y = \sec(x), \quad y = 0, \quad x = 0, \quad x = 1$$

About the x-axis.

<https://www.geogebra.org/m/tgceabb2#material/vte3zdjx>

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Example 3: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{6x} + \sqrt{\frac{x}{6}}, \quad x = 2, \quad x = 4$$

About the x-axis.

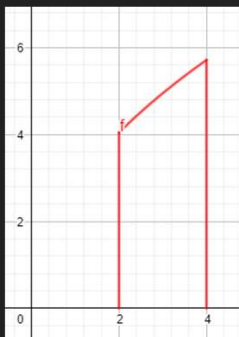
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Example 3: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{6x} + \sqrt{\frac{x}{6}}, \quad x = 2, \quad x = 4$$

About the x-axis.

First draw the region.



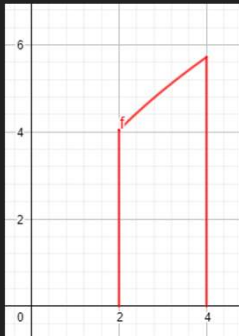
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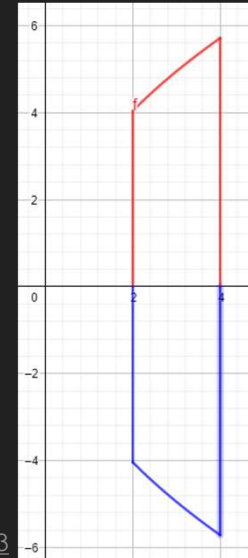
Example 3: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{6x} + \sqrt{\frac{x}{6}}, \quad x = 2, \quad x = 4$$

About the x-axis.



Rotation about x-axis



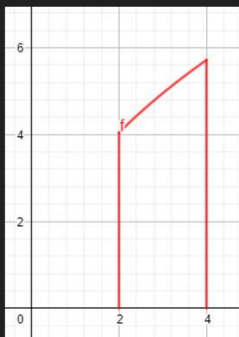
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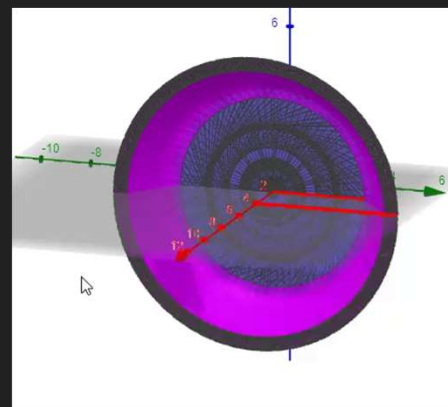
Example 3: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{6x} + \sqrt{\frac{x}{6}}, \quad x = 2, \quad x = 4$$

About the x-axis.



Furthermore, 3-D



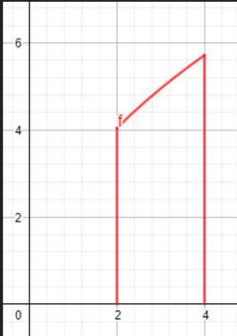
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Example 3: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{6x} + \sqrt{\frac{x}{6}}, \quad x = 2, \quad x = 4$$

About the x-axis.



$$\begin{aligned} V &= \pi \int_2^4 \left( \sqrt{6x} + \sqrt{\frac{x}{6}} \right)^2 dx \\ &= \pi \int_2^4 \left( (\sqrt{6x})^2 + 2\sqrt{6x} \sqrt{\frac{x}{6}} + \left( \sqrt{\frac{x}{6}} \right)^2 \right) dx \\ &= \pi \int_2^4 \left( 6x + 2x + \frac{x}{6} \right) dx \end{aligned}$$

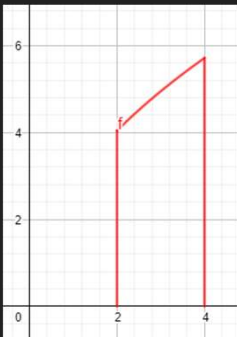
<https://www.geogebra.org/m/tgceabb2#material/njkxvte3>

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Example 3: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{6x} + \sqrt{\frac{x}{6}}, \quad x = 2, \quad x = 4$$

About the x-axis.



$$\begin{aligned} V &= \pi \int_2^4 \frac{49x}{6} dx \\ &= \pi \left[ \frac{49}{6} \frac{x^2}{2} \right]_2^4 \\ &= 49\pi \end{aligned}$$

<https://www.geogebra.org/m/tgceabb2#material/njkxvte3>

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Example 4: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = 4x^2, \quad x = 0, \quad y = 4$$

About the y-axis.

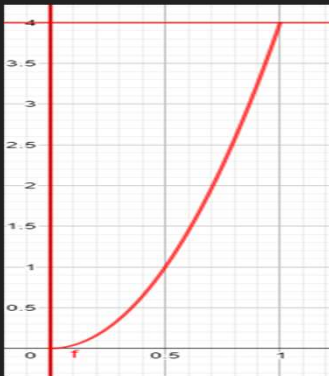
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Example 4: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = 4x^2, \quad x = 0, \quad y = 4$$

About the y-axis.

First draw the region.



<https://www.geogebra.org/m/tgceabb2#material/afywnvhr>

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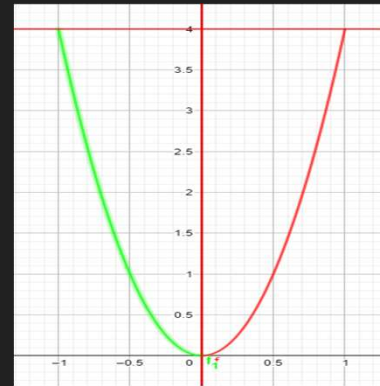
Example 4: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = 4x^2, \quad x = 0, \quad y = 4$$

About the y-axis.



Rotation about y-axis



<https://www.geogebra.org/m/tgceabb2#material/afywnvhr>

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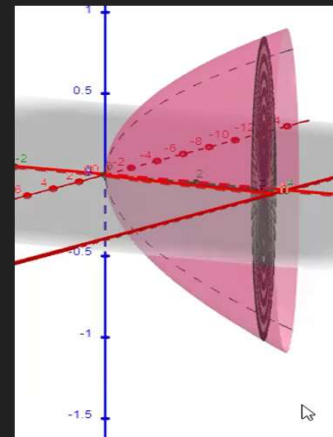
Example 4: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = 4x^2, \quad x = 0, \quad y = 4$$

About the y-axis.



Furthermore, 3-D



<https://www.geogebra.org/m/tgceabb2#material/afywnvhr>

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Example 4: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = 4x^2, \quad x = 0, \quad y = 4$$

About the y-axis.

First rewrite  $\uparrow$  as  $x =$



$$\frac{y}{4} = x^2$$

$$\sqrt{\frac{y}{4}} = x$$

Note there is no negative b/c we are in the 1st quadrant

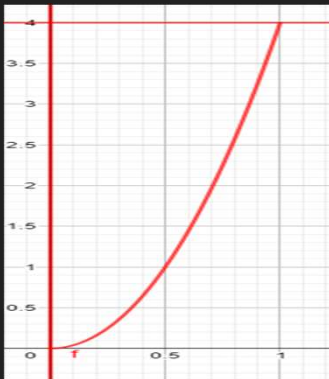
<https://www.geogebra.org/m/tgceabb2#material/afywnvhr>

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Example 4: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = 4x^2, \quad x = 0, \quad y = 4$$

About the y-axis.



$$V = \pi \int_0^4 \left( \sqrt{\frac{y}{4}} \right)^2 dy$$

$$= \pi \int_0^4 \frac{y}{4} dy$$

$$= \pi \left[ \frac{1}{4} \frac{y^2}{2} \right]_0^4$$

$$= 2\pi$$

<https://www.geogebra.org/m/tgceabb2#material/afywnvhr>

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Example 5: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{144 - x^2}, \quad x = 0, \quad y = 0$$

About the y-axis.

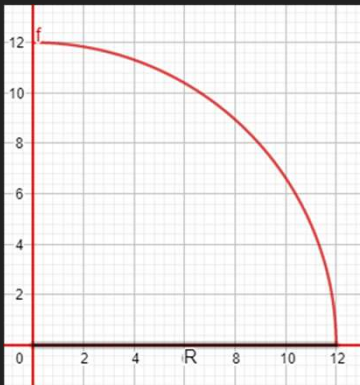
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Example 5: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{144 - x^2}, \quad x = 0, \quad y = 0$$

About the y-axis.

First draw the region.



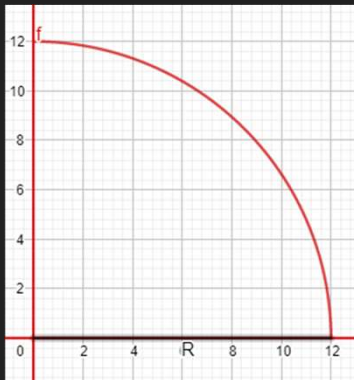
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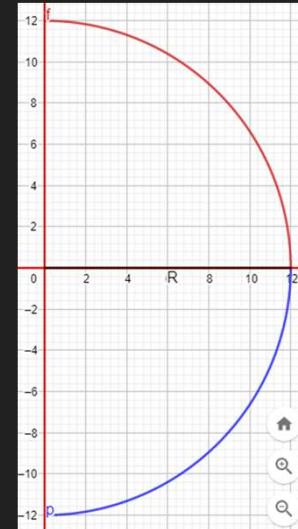
Example 5: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{144 - x^2}, \quad x = 0, \quad y = 0$$

About the y-axis.



Rotation about y-axis



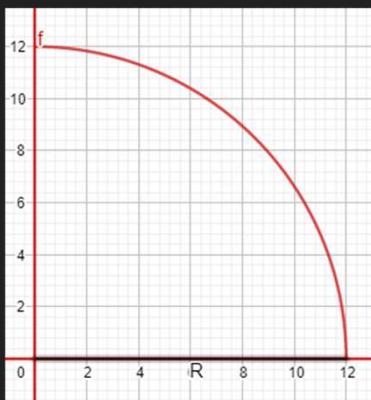
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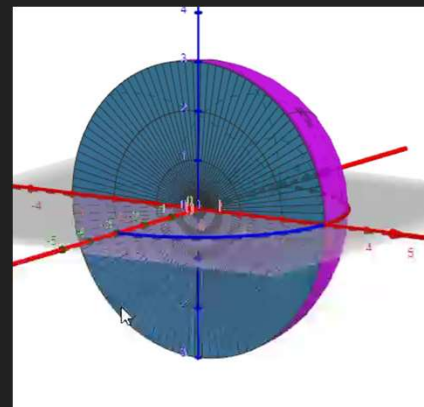
Example 5: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{144 - x^2}, \quad x = 0, \quad y = 0$$

About the y-axis.



Furthermore, 3-D



<https://www.geogebra.org/m/tgceabb2#material/a5s4n8u7>

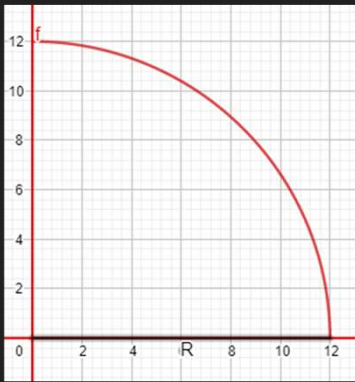
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Example 5: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{144 - x^2}, \quad x = 0, \quad y = 0$$

About the y-axis.

First rewrite  $\int$  as  $x =$



$$y^2 = 144 - x^2$$

$$144 - y^2 = x^2$$

$$44 - y^2$$

Note there is no negative b/c we are in the 1st quadrant

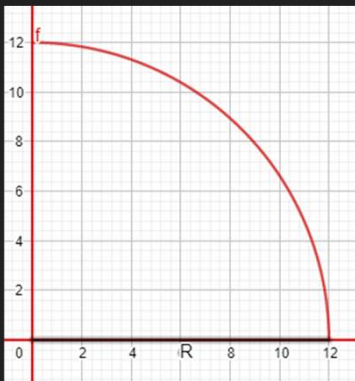
<https://www.geogebra.org/m/tqceabb2#material/a5s4n8u7>

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Example 5: Find the volume of the solid that results by revolving the region enclosed by the curves

$$y = \sqrt{144 - x^2}, \quad x = 0, \quad y = 0$$

About the y-axis.



$$V = \pi \int_0^3 (\sqrt{144 - x^2})^2 dx$$

$$= \pi \int_0^3 (144 - x^2) dx$$

$$= \pi \left( 144x - \frac{x^3}{3} \right) \Big|_0^3$$

$$= 423\pi$$

<https://www.geogebra.org/m/tqceabb2#material/a5s4n8u7>

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## GeoGebra Link for Lesson 14

○ <https://www.geogebra.org/m/tgceabb2>

○ Note click on the play buttons on the left-most screen and the animation will play/pause.