# MA 16020 LESSON 26: INTRODUCTION TO FUNCTIONS OF SEVERAL VARIABLES (ALGEBRA REVIEW)

## DOMAIN & RANGE OF SINGLE VARIABLE FUNCTIONS

**Recall the following common Domains and Ranges:**

1. $y = e^x$      **Domain:** $(-\infty, \infty)$      **Range:** $(0, \infty)$
2. $y = ln(x)$      **Domain:** $(0, \infty)$      **Range:** $(-\infty, \infty)$

> Note that $y = e^x$ and $y = ln(x)$ are inverses of each other. Which mean the domain of the first function is the range of the second (and vice versa).

3. $y = \sqrt{x}$      **Domain:** $[0, \infty)$      **Range:** $(-\infty, \infty)$
4. $y = \sqrt[3]{x}$      **Domain:** $(-\infty, \infty)$      **Range:** $(-\infty, \infty)$

> Note: Let $y = \sqrt[n]{x} = x^{1/n}$.
> - If $n$ is even, then    **Domain:** $[0, \infty)$      **Range:** $(-\infty, \infty)$
> - If $n$ is odd, then    **Domain:** $(-\infty, \infty)$      **Range:** $(-\infty, \infty)$

## Techniques for finding the Domain:

- Given $\sqrt{?}$ then $? \geq 0$
- Given $ln\ ?$ then $? > 0$

- Given $\dfrac{1}{?}$ then $? \neq 0$
- Given $\dfrac{1}{\sqrt{?}}$ then $? > 0$

**Exercise 1:** Find the Domain and Range of the following:

1. $y = \sqrt{2x + 3}$ 

   **Domain:** 

   **Range:**

2. $y = \sqrt{x^2 - 1}$ 

   **Domain:** 

   **Range:**

3. $y = \ln(x^2 + 2x + 1)$ 

   **Domain:** 

   **Range:**

4. $y = \dfrac{1}{x+4}$ 

   **Domain:** 

   **Range:**

5. $y = \dfrac{1}{\sqrt{5x+1}}$ 

   **Domain:** 

   **Range:**

6. $y = \dfrac{\sqrt{x-1}}{x^2+3x-4}$ 

   **Domain:** 

   **Range:**

7. $y = \dfrac{\sqrt{2x-1}}{\ln(10x-5)}$ 

   **Domain:** 

   **Range:**

8. $y = \sqrt[4]{7x + 4}$ 

   **Domain:** 

   **Range:**

9. $y = \dfrac{\ln(x+2)\sqrt[4]{2x+1}}{\sqrt{x-6}}$ 

   **Domain:** 

   **Range:**

1. Point at the origin     $\Rightarrow$     $(0,0)$

2. Lines     $\Rightarrow$     $y = mx + b$        where $m$ is the slope and $b$ is the y-intercept

3. Parabolas     $\Rightarrow$     $y = a(x - h)^2 + k$     where $(h, k)$ is the vertex of the parabola

4. Exponential Functions
   a. Increasing     $\Rightarrow$     example    $y = e^x$
   b. Decreasing     $\Rightarrow$     example    $y = e^{-x}$

5. Logarithmic Functions
   a. Increasing     $\Rightarrow$     example    $y = \ln x$
   b. Decreasing     $\Rightarrow$     example    $y = -\ln x$

6. Rational Functions are functions of the form: $y = \dfrac{p(x)}{q(x)}$
   a. x-axis symmetry                    b. y-axis symmetry
       $\Rightarrow$     $f(x) = -f(x)$                    $\Rightarrow$     $f(x) = f(-x)$
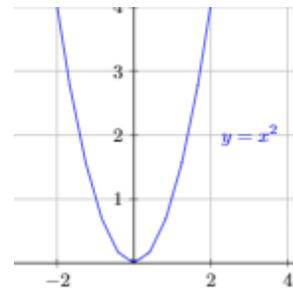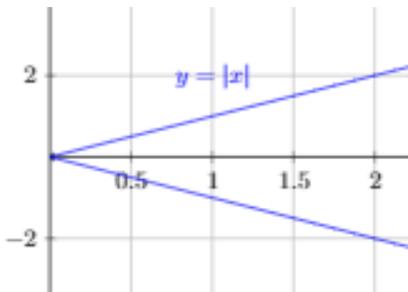
7. Circles     $\Rightarrow$     $(x - h)^2 + (y - k)^2 = r^2$    where $r$ is radius and $(h, k)$ is the center

8. Ellipses     $\Rightarrow$     $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$     where $(h, k)$ is the center

9. Hyperbolas     $\Rightarrow$     $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$     where $(h, k)$ is the center

To find the foci for 8 and 9, we use the equation $c^2 = a^2 + b^2$, and solve for $c$.