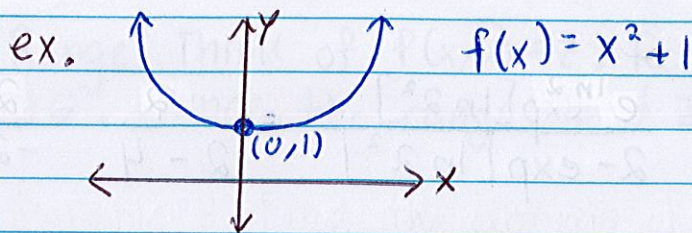


# Lesson 26: Intro to Functions of Several Variables

## Single Variable Functions

- Can be written as  $y = f(x)$   
i.e.  $y$  is a function of  $x$
- Takes as an input a # and produces as an output a #
- Graph: a curve in the  $xy$ -plane  
i.e. 2-D

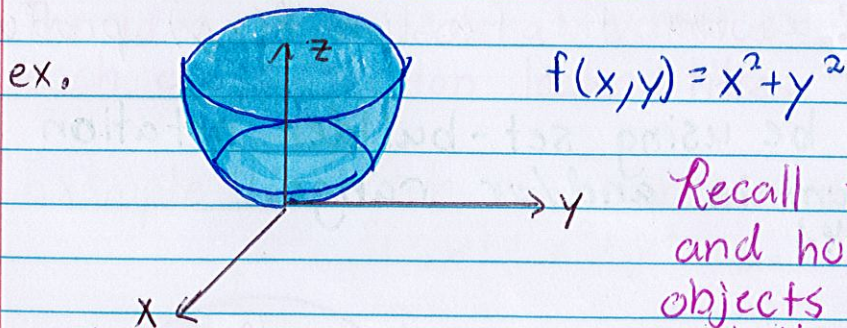


## Multivariable Functions

Definition: A multivariable function has 2 or more variables as inputs.

In this class, we are limiting ourselves to just 2 variables. So

- Can be written as  $z = f(x, y)$   
i.e.  $z$  is a function of  $x$  and  $y$
- Takes as an input a pair of # or a point and produces as an output a #
- Graph: A surface in  $xyz$ -space  
i.e. 3-D



Recall the idea of cross sections and how stacking multiple 2-D objects became one 3-D object. Well this is a bunch of circles with increasing radii stacked on top of another.

Example 1: Compute the indicated functional value

$$(a) f(x,y) = \frac{3x+2y}{2x+3y} ; f(-4,6)$$

$$f(-4,6) = \frac{3(-4)+2(6)}{2(-4)+3(6)} = \frac{-12+12}{-8+18} = \frac{0}{10} = 0$$

$$(b) f(x,y) = \frac{e^{xy}}{2-e^{xy}} ; f(\ln 2, 2) \text{ and } f(1, \ln 1)$$

$$f(\ln 2, 2) = \frac{e^{2 \ln 2}}{2 - e^{2 \ln 2}} = \frac{\exp[\ln 2^2]}{2 - \exp[\ln 2^2]} = \frac{4}{2-4} = \frac{4}{-2} = -2$$

$$f(1, \ln 1) = \frac{e^{\ln 1}}{2 - e^{\ln 1}} = \frac{1}{2-1} = \frac{1}{1} = 1$$

### Domain and Range for Multivariable Functions

Just as for functions of one variables, we can find the domain and range of functions of two variable in a similar fashion. **If you need a review of domain and range of one variable functions, look at the Algebra Review pdf for Lesson 18.**

The main difference for two variable functions is

Domain: All points  $(x,y)$  in the  $xy$ -plane for which  $f(x,y)$  is defined

Range: All values that the function  $f(x,y)$  produces

Notation: We will be using set-builder notation for denoting the domain and/or range.

i.e.  $\{x \mid x \geq 4\}$

Example 2: Describe the domain and range of the function  
 $f(x,y) = \sqrt{x^2 - y}$

Domain: Recall domain of  $\sqrt{\cdot}$  is  $\cdot \geq 0$ . So

$$x^2 - y \geq 0$$

$$x^2 \geq y$$

Hence the domain is  $\{(x,y) \mid x^2 \geq y\}$ . Note this domain consists parabolas.

Range: Think of  $f(x,y) = z$ . Recall that the range of  $\sqrt{\cdot}$  is  $\geq 0$ . Hence the range is  $\{z \mid z \geq 0\}$

Example 3: Find the domain of  $f(x,y) = \frac{\sqrt{x-18}}{\ln(y-9)}$

To find the domain, we need to play with **3** portions of the function

$$\begin{aligned} \bullet \sqrt{x-18} &\Rightarrow x-18 \geq 0 & \bullet \ln(y-9) &\Rightarrow y-9 > 0 & \bullet \frac{1}{\ln(y-9)} &\Rightarrow \ln(y-9) \neq 0 \\ &x \geq 18 & &y > 9 & &y-9 \neq e^0 \\ & & & & &y-9 \neq 1 \\ & & & & &y \neq 10 \end{aligned}$$

Hence the domain is  $\{(x,y) \mid x \geq 18, y > 9, y \neq 10\}$

Example 3': Find the domain of  $f(x,y) = \frac{\sqrt{x-18}}{\ln(y-9)-3}$

To find the domain, we need to play with 3 portions of the function.

$$\begin{aligned} \bullet \sqrt{x-18} &\Rightarrow x-18 \geq 0 & \bullet \ln(y-9) &\Rightarrow y-9 > 0 \\ &x \geq 18 & &y > 9 \end{aligned}$$

$$\begin{aligned} \bullet \frac{1}{\ln(y-9)-3} &\Rightarrow \ln(y-9)-3 \neq 0 \\ &\ln(y-9) \neq 3 \\ &y-9 \neq e^3 \\ &y \neq 9+e^3 \end{aligned}$$

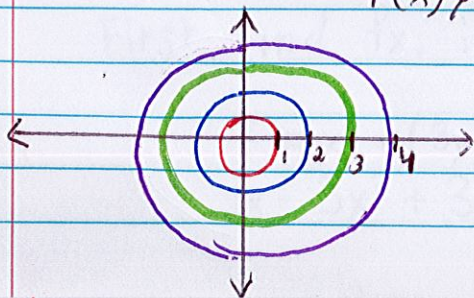
Hence the domain is  $\{(x,y) \mid x \geq 18, y > 9, y \neq 9+e^3\}$

## Level Curves

Definition: The level curves of a function of two variables are the curves with equations  $f(x,y) = k$  where  $k$  is a constant (in the range of  $f$ ).

The idea is to look at various values of  $z$  and see what each cross-section looks like.

Example 4: Sketch the indicated level curves  
 $f(x,y) = x^2 + y^2, C = 1, 4, 9, 16$



$$\begin{aligned}x^2 + y^2 &= 1 = 1^2 \\x^2 + y^2 &= 4 = 2^2 \\x^2 + y^2 &= 9 = 3^2 \\x^2 + y^2 &= 16 = 4^2\end{aligned}$$

Example 5: What do the level curves for  $f(x,y) = 11\sqrt{y+6x^2}$  look like?

Let  $f(x,y) = k$ . So  $k = 11\sqrt{y+6x^2}$ . Now solve for  $y$ .

$$\frac{k}{11} = \sqrt{y+6x^2}$$

$$\left(\frac{k}{11}\right)^2 = y+6x^2$$

$$\frac{k^2}{121} - 6x^2 = y$$

$\downarrow$   
constant

So the level curves are in the shapes of parabolas.

