## Lesson 27. Partial Derivatives

A partial derivative is a derivative where we hold some variables constant.

Let's think about a function of one variable. ex.  $f(x)=x^2 \implies f'(x)=2x$ 

But what about a function of two variables? f(x,y)= x2+y

We find its partial derivative with respect to x by treating y as a constant.  $f_x = 2x + 0^2 2x$ 

To find the partial clerivative with respect to y, we treat x as a constant.  $fy = 0 + 3y^2 = 3y^2$ 

Definition: The (first) partial derivative fx describes the rate of change of f as x changes, where y remains constant. i.e. Find the derivative with respect to x, where we treat y as a constant

· The (first) partial derivative fy describes the rate of change of f as y changes, where x remains constant. I.e. Find the derivative with respect to y, where we treat x as a constant.

Example 1: Compute the first order partial derivatives  $\Theta f(x,y) = x^3 + 3xy$ 

First order partials => We need to find fx and fy.

First find fx. i.e. Find the derivative w/ respect to x and treat y as a constant.

 $f = x^3 + (3y)x$   $f_{x} = 3x^2 + 3y$ 

	Southerned Harriso P. March 201
	Next find for her Find the derivative w/ respect to a and
zadamov	treat x as a constant.
	Next find fy. i.e. Find the derivative w/ respect to y and treat x as a constant. $f= x^3 + (3x)y$ $f_y=0+3x=3x$
	$f_{y=0} + 3x = 3x$
Chain	(2) P(x 1) = 1n/x + 2)
Rule	First find fx. i.e. Find the derivative w/ respect to x
Problem	First find fx, i.e. Find the derivative w/ respect to x and treat y as a constant, $ \frac{f_x}{x+2y} = \frac{d}{dx}(x+2y) = \frac{1}{x+2y} \cdot (1+0) = \frac{1}{x+2y} $
	$f_{x} = \frac{1}{1000} \cdot \frac{d}{d}(x+2y) = \frac{1}{1000} \cdot (1+0) = \frac{1}{1000}$
	X+dy dx X+dy X+dy
2.7 (1	Next find fv. i.e. Find the derivative w/ respect to y and
	Next find fy. i.e. Find the derivative w/ respect to y and treat x as a constant,
	$\frac{fy^2}{x+2y} \cdot \frac{d}{dy} (x+2y)^2 \frac{1}{x+2y} \cdot (0+2)^2 \frac{2}{x+2y}$
	x+dy dy x+dy x+dy
	$\bigcirc f(x,y) = \frac{qxy}{\sqrt{y-1}}$
<u> </u>	Definition of the first partial derivity for description
cun 5 Vulnezuer	First find fx, i.e. Find the derivative w/ respect to x and treat y as a constant,
313410	f(x,y)= 9y (y)
	Jy-17 (1)
ડ ફેલ્લ્ડ	$f_{X} = \frac{q_{Y}}{Q_{X}} \cdot \frac{d(x)}{Q_{X}} = \frac{q_{Y}}{Q_{X}}$
enselva .	y may y -1 of dx
	Next find fy, i.e. Find the derivative w/ respect to y and
	treat x as a constant,
	$P(x,y) = q_x \left( \frac{y}{y} \right)$
apply	Py=9x d ( y)=9x/10/y-1'-y.1/2(y-1)-1/2)
Quitent	$dy(\overline{y-1})$ $(\overline{y-1})^2$
Rule	$= 9 \times (\sqrt{y-1} - 1/2 \times (y-1)^{-1/2})$
	11-V 03 6 (60) Stant.
	\\2 + 3x <sup>2</sup> + 3y

Example 2: Evaluate the partial derivatives  $f_x(x,y)$  and  $f_y(x,y)$  at the given point  $P_o(x_0,y_0)$ ,  $f(x,y) = x^3y^2 + 6x^2$ ;  $P_o(1,-1)$ First find fx, i.e. Find the derivative w/ respect to x and treat y as a constant,

fx = 3x2y2+12x2 Plug (1,-1) into fx.  $f_X(1,-1) = 3(1)^2(-1)^2 + 12(1)^2 = 15$ Next find fy, i.e. Find the derivative w/ respect to y and treat x as a constant, fy =  $x^3 \cdot 2y = 2x^3y$ Plug (1,-1) into fy.  $f_y(1,-1) = 2(1)^3(-1) = -2$