Lesson 8: Integration by Parts II

Recall from Last time, we found the formula for Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

We were also given an acronym to help in choosing what u should be.

L - Logarithmic
A - Algebraic (like polynomials)
T - Trigonometric
E - Exponential

Example 1: Evaluate

ⓐ $\int (t-3)e^t\,dt \quad \dfrac{u = t-3 \qquad dv = e^t\,dt}{du = dt \qquad v = e^t} \quad uv - \int v\,du$

$$= (t-3)e^t - \int e^t\,dt = (t-3)e^t - e^t + C$$

$$= (t-3-1)e^t + C = (t-4)e^t + C$$

ⓑ $\int \ln x\,dx \quad \dfrac{u = \ln x \qquad dv = dx}{du = \frac{1}{x}dx \qquad v = x} \quad uv - \int v\,du$

$$= x\ln x - \int x \cdot \frac{1}{x}\,dx = x\ln x - \int dx$$

$$= x\ln x - x + C$$

ⓒ $\int x^2 \sin x\,dx \quad \dfrac{u = x^2 \qquad dv = \sin x\,dx}{du = 2x\,dx \qquad v = -\cos x} \quad uv - \int v\,du$

$$= -x^2\cos x - \int(-\cos x)2x\,dx$$

$$= -x^2\cos x + \underbrace{\int 2x\cos x\,dx}$$

Note to integrate this we need to do another integration by parts

$$\int 2x\cos x\,dx \quad \frac{u=2x}{du=2dx} \quad \frac{dv=\cos x\,dx}{v=\sin x} \quad uv-\int v\,du$$

$$= 2x\sin x - \int 2\sin x\,dx = 2x\sin x - (-2\cos x)$$

$$= 2x\sin x + 2\cos x$$

$$= x^2\cos x + 2x\sin x + 2\cos x + C$$

Example 2: The velocity of a car over the time period $0 \le t \le 3$ is given by the function

$$v(t) = 50te^{-t/4} \text{ miles per hour}$$

where t is time in hours. What was the distance the car traveled in the first 30 minutes?

Note t is in hours. So 30 mins $\Rightarrow$ 0.5 hrs

We need to compute

$$\int_0^{0.5} 50te^{-t/4}\,dt \quad \frac{u=50t}{du=50dt} \quad \frac{dv=e^{-t/4}\,dt}{v=-4e^{-t/4}} \quad uv-\int v\,du$$

$$= 50t(-4e^{-t/4})\Big]_0^{0.5} - \int_0^{0.5} -4e^{-t/4}(50)\,dt$$

$$= -200te^{-t/4}\Big]_0^{0.5} + 200\int_0^{0.5} e^{-t/4}\,dt$$

$$= -200te^{-t/4}\Big]_0^{0.5} + 200(-4)e^{-t/4}\Big]_0^{0.5}$$

$$= -200(0.5)e^{-0.5/4} - (-200(0)e^{-0/4}) - 800e^{-0.5/4}$$

$$- (-800e^{-0/4})$$

$$= -100e^{-1/8} - 0 - 800e^{-1/8} + 800$$

$$= -900e^{-1/8} + 800$$

$$\approx 5.75$$

Example 3: A model for the ability of a child to memorize information, measured on a scale from 1 to 100, is given by

$$M(t) = 1 + 3.4t\ln(t)$$

when $2 \le t \le 8$ where t is the child's age in years. Find the child's average memorization ability between ages 3 and 6.

$$M_{AVE}(t) = \frac{1}{6-3}\int_3^6 (1 + 3.4t\ln(t))\,dt$$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\int_3^6 t\ln(t)\,dt$$

$u = \ln(t)$	$dv = t\,dt$
$du = \frac{1}{t}\,dt$	$v = \frac{t^2}{2}$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\left(\frac{t^2}{2}\ln(t)\Big]_3^6 - \int_3^6 \frac{t^2}{2}\cdot\frac{1}{t}\,dt\right)$$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\left(\frac{t^2}{2}\ln(t)\Big]_3^6 - \int_3^6 \frac{1}{2}t\,dt\right)$$

$$= \frac{1}{3}t\Big]_3^6 + \frac{3.4}{6}t^2\ln(t)\Big]_3^6 - \frac{3.4}{6}\cdot\frac{t^2}{2}\Big]_3^6$$

$$= \frac{1}{3}(6) - \frac{1}{3}(3) + \frac{3.4}{6}(6)^2\ln(6) - \frac{3.4}{6}(3)^2\ln(3)$$

$$- \frac{3.4}{12}(6)^2 - \left(\frac{-3.4}{12}(3)^2\right)$$

$$\approx 26.299$$