MATH 16020: Applied Calculus 2 Quiz 10 (Solutions) Fri., Nov. 11, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:______________________________

1. **[4 pts]** Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

Solution: To find the domain of $f(x,y)$, we need to solve the following inequalities:

- **[1 pt]** $\sqrt{x+y-1} \;\Rightarrow\; x+y-1 \geq 0$
  $y \geq 1-x$
- **[1 pt]** $\ln(y-11) - 9 \neq 0$
  $\ln(y-11) \neq 9$
  $y - 11 \neq e^9$
  $y \neq e^9 + 11$
- **[1 pt]** $\ln(y-11) \;\Rightarrow\; y-11 > 0$
  $y > 11$

Hence when we put them all together, the domain is

$$\{(x,y) \mid y \geq 1-x,\;\; y > 11,\;\; y \neq e^9+11\}$$ **[1 pt]**

2. **[2 pts]** Describe the indicated level curves $f(x,y) = C$

$$f(x,y) = \ln(x^2+y^2); \qquad C = \ln(36)$$

(a) Parabola with vertices at $(0,0)$
(b) Circle with center at $(0, \ln 36)$ and radius 6
(c) Parabola with vertices at $(0, \ln 36)$
(d) Circle with center at $(0,0)$ and radius 6
(e) Circle with center at $(0,0)$ and radius $\ln(36)$
(f) Increasing Logarithm Function

Solution: As the problem statement states, we are looking at

$$f(x,y) = C$$
$$\ln(x^2+y^2) = \ln(36)$$

Note if we take e on both sides, we get

$$x^2+y^2 = 36 = 6^2$$

which is the equation of a circle center at $(0,0)$ and radius 6.

3. **[4 pts]** Compute $f_x(6,5)$ when

$$f(x,y) = \frac{(6x-6y)^2}{\sqrt{y-1}}$$

Solution: Note that we want f_x. So y is a constant. Hence we can do the following rewrite:

$$f(x,y) = \frac{1}{\sqrt{y-1}} \cdot (6x-6y)^2$$

So,

$$f_x(x,y) = \frac{d}{dx}\left(\frac{1}{\sqrt{y-1}} \cdot (6x-6y)^2\right) = \frac{1}{\sqrt{y-1}} \cdot \frac{d}{dx}\left((6x-6y)^2\right)$$

since again y is a constant. By chain rule,

$$f_x(x,y) = \frac{1}{\sqrt{y-1}} \cdot \frac{d}{dx}\left((6x-6y)^2\right) = \frac{1}{\sqrt{y-1}} \cdot (2(6x-6y)6) = \frac{12(6x-6y)}{\sqrt{y-1}} \qquad \textbf{[3 pts]}$$

Hence

$$f_x(6,5) = \frac{12(6(6)-6(5))}{\sqrt{5-1}} = 36 \qquad \textbf{[1 pt]}$$