Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:____

1. **[4 pts]** Suppose that $f(x,y)$ can be written as a product $f(x,y) = F(x)G(y)$ of a function of x and a function of y. Then the integral of f over the rectangle R: $a \le x \le b$, $c \le y \le d$ can be evaluated as a product as well, by the formula

$$\iint\limits_R f(xy)\,dA = \left(\int_a^b F(x)\,dx\right)\left(\int_c^d G(y)\,dy\right)$$

Provide a justification steps (i) through (iv) of the following argument.

$$\iint\limits_R f(x,y)\,dA = \int_c^d \left(\int_a^b F(x)G(y)\,dx\right) dy \qquad (i) \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$= \int_c^d \left(G(y)\int_a^b F(x)\,dx\right) dy \qquad (ii) \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$= \int_c^d \left(\int_a^b F(x)\,dx\right) G(y)\,dy \qquad (iii) \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$= \left(\int_a^b F(x)\,dx\right)\left(\int_c^d G(y)\,dy\right) \qquad (iv) \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

2. **[2 pts]** For what values of the constant k does the second derivative test guarantee that

$$f(x, y) = x^2 + kxy + y^2$$

will have a saddle point at $(0, 0)$? **Give reasons for your answers.**

$k =$ ______________________

Why? ______________________

[2 pts] A local minimum at $(0, 0)$? **Give reasons for your answers.**

$k =$ ______________________

Why? ______________________

[2 pts] For what values of k is the second derivative test inconclusive? **Give reasons for your answers.**

$k =$ ______________________

Why? ______________________

The End.