MATH 16020: Applied Calculus 2 Quiz 7 (Solutions) Mon., Oct. 16, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

1. **[5 pts]** The radioactive isotope ^{226}Ra has a half-life of 1,599 years. If there are 10 grams of ^{226}Ra initially, how much is there are 1,000 years.

   (Round your answer to 4 decimal places)

   Solution: Recall that half-life problems have:

   $$y = Ce^{kt} \qquad \text{and} \qquad k = \frac{-ln(2)}{\text{half-life}} \qquad \textbf{[2 pts]}$$

   [1 pt] So $k = \dfrac{-\ln(2)}{\text{half-life}} = \dfrac{-ln(2)}{1599}$ and $C = 10$. So,

   $$y = 10\exp\left[\frac{-\ln(2)}{1599}t\right] \qquad \textbf{[1 pt]}$$

   [1 pt] So, $y(1000) \approx 6.4828$ grams.

2. **[5 pts]** The rate of change of the population $N(t)$ of a sample of bacteria is directly proportional to the number of bacteria present, so

   $$N'(t) = kN$$

   where time t is measured in hours. Initially, there are 200 bacteria present. If the number of bacteria after 3 hours is 400, find the growth rate k in terms of hours.

   (Round your answer to 3 decimal places)

   Solution: Recall that $N'(t) = kN$ means that

   $$N(t) = Ce^{kt} \qquad \textbf{[1 pt]}$$

   When $N(0) = 200$,

   $$200 = N(0) = Ce^{k\cdot 0} = C \qquad \textbf{[1 pt]}$$

   So, $N(t) = 200e^{kt}$. When $N(3) = 400$,

   $$400 = N(3) = 200e^{3k} \qquad \textbf{[1 pt]}$$

   $$2 = \frac{400}{200} = e^{3k}$$

   $$\ln(2) = 3k \qquad \textbf{[1 pt]}$$

   $$k = \frac{1}{3}\ln(2) \approx 0.231 \qquad \textbf{[1 pt]}$$