MATH 16020: Applied Calculus 2 Quiz 8 (Solutions) Wed., Oct. 26, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:____

1. **[2 pts]** Circle the technique you think is easiest to find the general solution of the differential equation:

$$\frac{dy}{dt} = -\frac{2}{3}y$$

Don't Solve!

A) Exponential Growth and Decay Formula

B) Separable Functions

C) First-Order Linear Equations

Solution: The easiest is technique to use is A) Exponential Growth and Decay Formula, which is

$$y' = ky \qquad \Rightarrow \qquad y = Ce^{kt}$$

Hence $y = Ce^{-(2/3)t}$.

2. **[4 pts]** What is the **integrating factor** of the following differential equation?

$$y' + (\cot(x))y = \sin^2(x)$$

Solution: Note that the differential equation is already in Standard Form.

So $P(x) = \tan(x)$ **[1 pt]**.

$$\begin{aligned} u(x) = \exp\left[\int P(x)\,dx\right] &= \exp\left[\int \cot(x)\,dx\right] \\ &= \exp\left[\int \frac{\cos(x)}{\sin(x)}\,dx\right] \\ &\underset{=\cos(x)\,dx}{\overset{u=\sin(x)}{=\!=\!=}} \exp\left[\int \frac{1}{u}\,du\right] = \exp\left[\ln(u)\right] = u = \sin(x) \qquad \textbf{[3 pts]} \end{aligned}$$

3. [**4 pts**] What is the **integrating factor** of the following differential equation?

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = \frac{e^{x^2}}{x+1}$$

Solution: First get the differential equation is already in Standard Form. i.e.

$$\frac{dy}{dx} - \frac{2(x^2+x)}{x+1}y = \frac{e^{x^2}}{(x+1)^2} \qquad \textbf{[1 pt]}$$

So $P(x) = -\dfrac{2(x^2+x)}{x+1} = -\dfrac{2x(x+1)}{x+1} = -2x$ [**1 pt**].

$$u(x) = \exp\left[\int P(x)\,dx\right] = \exp\left[\int -2x\,dx\right] = \exp\left[-x^2\right] = e^{-x^2} \qquad \textbf{[2 pts]}$$