MATH 16020: Applied Calculus 2 Quiz 9 (Solutions) Wed., Nov. 2, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:________________________

1. **[2 pts each]** If the given series converges, then find its sum. If not, state that it diverges.

(a) $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

Solution: [2 pts] Since our sum starts at $n = 0$, we can check whether our r satisfies the condition:

$$|r| < 1$$

Since $r = 3/2$, it doesn't satisfy the condition. Hence the series diverges.

(b) $\sum_{n=0}^{\infty} 6\left(\frac{-1}{9}\right)^n$

Solution: [1 pt] Since our sum starts at $n = 0$, we can check whether our r satisfies the condition:

$$|r| < 1$$

Since $r = -1/9$, it does satisfy the condition.

Hence we can use the Geometric Series Formula

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n = \frac{6}{1-(-1/9)} = \frac{54}{10} \quad \textbf{[1 pt]}$$

2. **[6 pts]** Express $f(x) = \dfrac{3}{1+2x}$ as a power series and it's radius of convergence.

Solution: Let's rewrite $f(x)$ to look like our Power Series Formula.

$$\frac{3}{1+2x} = 3 \cdot \frac{1}{1-(-2x)} \qquad \textbf{[1 pt]}$$

So replace x with $-2x$ in our formula.

$$\sum_{n=0}^{\infty} (-2x)^n = \frac{1}{1-(-2x)} \qquad \text{where } |-2x| < 1 \qquad \textbf{[2 pts]}$$

Now multiply both sides by 3.

$$3\sum_{n=0}^{\infty} (-2x)^n = \frac{3}{1-(-2x)} \qquad \text{where } 2 \cdot |x| < 1 \qquad \textbf{[1 pt]}$$

Hence

$$\sum_{n=0}^{\infty} 3(-1)^n 2^n x^n = \frac{3}{1+2x} \qquad \text{where } |x| < \frac{1}{2} \qquad \textbf{[2 pts]}$$

Hence the radius of convergence is $R = 1/2$.