# Lesson 11: Area Between Two Curves

**Bonus Material:** Though most of your homework problems can be done by integrating with respect to $x$, in the future this may not be the case.

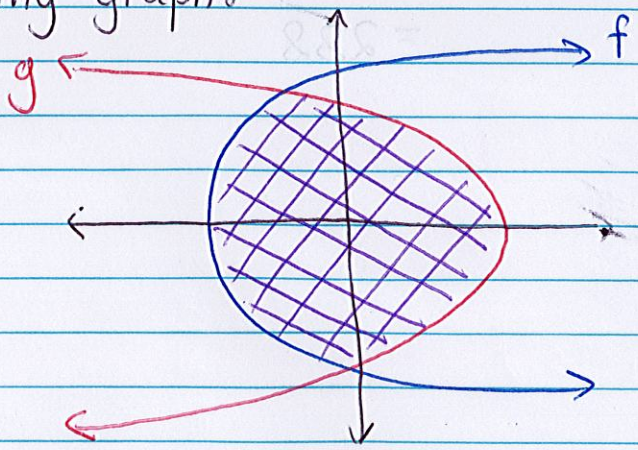
i.e. Some problems will be easier to do by integrating with respect to $y$.

How does this change the problem at hand?

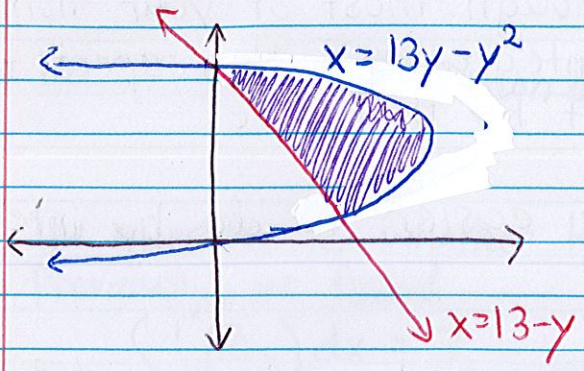
- The roles of $x$ and $y$ switch.
- Given two curves $f$ and $g$ which are functions of $y$, we want to integrate the one with larger $x$-values minus the one with the smaller $x$-values.
- We amend "Top minus Bottom" to "Bigger minus Smaller."

$$\text{Area} = \int_c^d \text{Right}\, dy - \int_c^d \text{Left}\, dy$$
$$= \int_c^d (F(y) - G(y))\, dy$$

An example where this technique may be useful is for the following graph:



**Bonus Example:** Find the area bounded by the curves $x = 13y - y^2$ and $x + y = 13$




From the graph,
Bigger $\Rightarrow$ $x = 13y - y^2$
Smaller $\Rightarrow$ $x = 13 - y$

So $\int_c^d (13y - y^2) - (13 - y)\,dy$

Let's find the intersection points.

$$13y - y^2 = 13 - y$$
$$0 = y^2 - 14y + 13$$
$$0 = (y - 13)(y - 1)$$
$$y = 13, \quad y = 1$$

Hence $\int_1^{13} (13y - y^2 - 13 + y)\,dy = \int_1^{13} (14y - y^2 - 13)\,dy$

$$= \left(\frac{14y^2}{2} - \frac{y^3}{3} - 13y\right)\Big]_1^{13}$$
$$= \left(7y^2 - \frac{y^3}{3} - 13y\right)\Big]_1^{13}$$
$$= 7(13)^2 - \frac{13^3}{3} - 13(13) - \left(7 - \frac{1}{3} - 13\right)$$
$$= 288$$