MA 16020 LESSON 18: INTRODUCTION TO FUNCTIONS OF SEVERAL VARIABLES (ALGEBRA REVIEW)

DOMAIN & RANGE OF SINGLE VARIABLE FUNCTIONS

Recall the following common Domains and Ranges:

1. $y = e^x$ **Domain:** $(-\infty, \infty)$ **Range:** $(0, \infty)$
2. $y = ln(x)$ **Domain:** $(\mathbf{0}, \infty)$ **Range:** $(-\infty, \infty)$

Note that $y = e^x$ and $y = ln(x)$ are inverses of each other. Which mean the domain of the first function is the range of the second (and vice versa).

3. $y = \sqrt{x}$ **Domain:** $[\mathbf{0}, \infty)$ **Range:** $(-\infty, \infty)$
4. $y = \sqrt[3]{x}$ **Domain:** $(-\infty, \infty)$ **Range:** $(-\infty, \infty)$

Note: Let $y = \sqrt[n]{x} = x^{1/n}$.

- If n is even, then **Domain:** $[\mathbf{0}, \infty)$ **Range:** $(-\infty, \infty)$
- If n is odd, then **Domain:** $(-\infty, \infty)$ **Range:** $(-\infty, \infty)$

Techniques for finding the Domain:

- Given $\sqrt{?}$ then $? \geq 0$
- Given $\frac{1}{?}$ then $? \neq 0$
- Given $ln\ ?$ then $? > 0$
- Given $\frac{1}{\sqrt{?}}$ then $? > 0$

Exercise 1: Find the Domain and Range of the following:

1. $y = \sqrt{2x + 3}$ **Domain:** **Range:**

2. $y = \sqrt{x^2 - 1}$ **Domain:** **Range:**

3. $y = ln(x^2 + 2x + 1)$ **Domain:** **Range:**

4. $y = \frac{1}{x+4}$ **Domain:** **Range:**

5. $y = \frac{1}{\sqrt{5x+1}}$ **Domain:** **Range:**

6. $y = \frac{\sqrt{x-1}}{x^2+3x-4}$ **Domain:** **Range:**

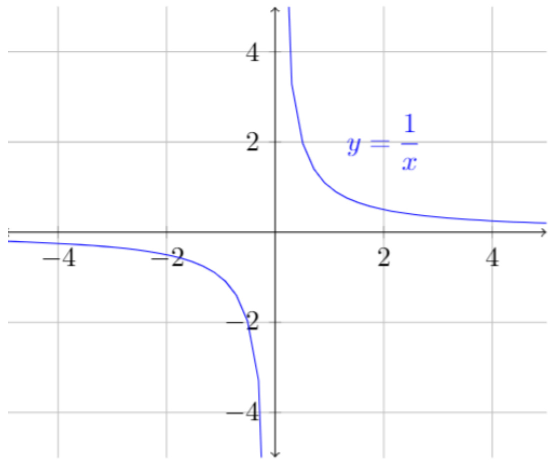
7. $y = \frac{\sqrt{2x-1}}{ln(10x-5)}$ **Domain:** **Range:**

8. $y = \sqrt[4]{7x + 4}$ **Domain:** **Range:**

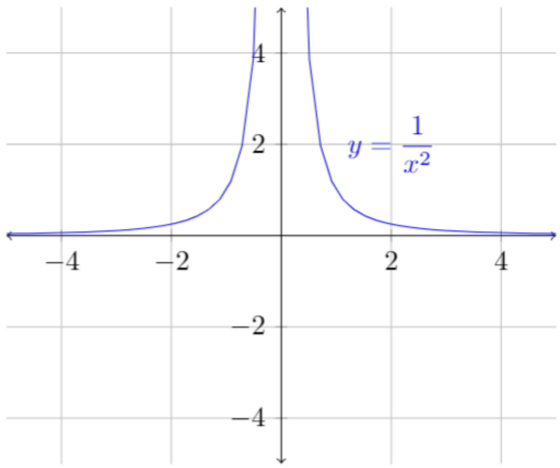
9. $y = \frac{ln(x+2)\sqrt[4]{2x+1}}{\sqrt{x-6}}$ **Domain:** **Range:**

VERTICAL + HORIZONTAL ASYMPTOTES

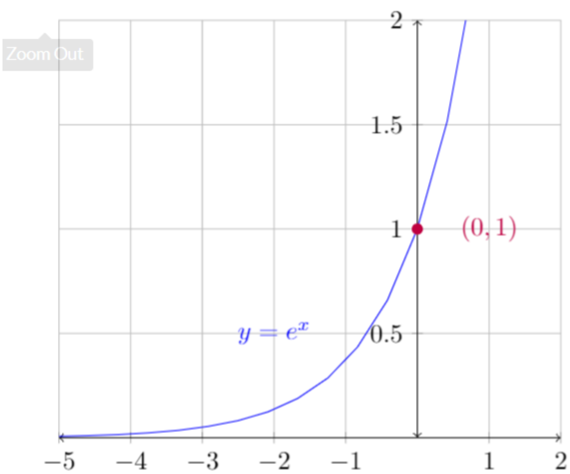
Recall the following functions:



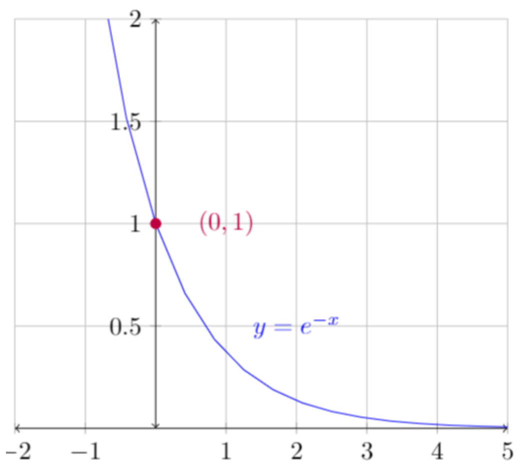
For $y = \frac{1}{x}$ HA: $y = 0$
VA: $x = 0$



For $y = \frac{1}{x^2}$ HA: $y = 0$
VA: $x = 0$



For $y = e^x$ HA: $y = 0$
VA: N/A



For $y = e^{-x}$ HA: $y = 0$
VA: N/A

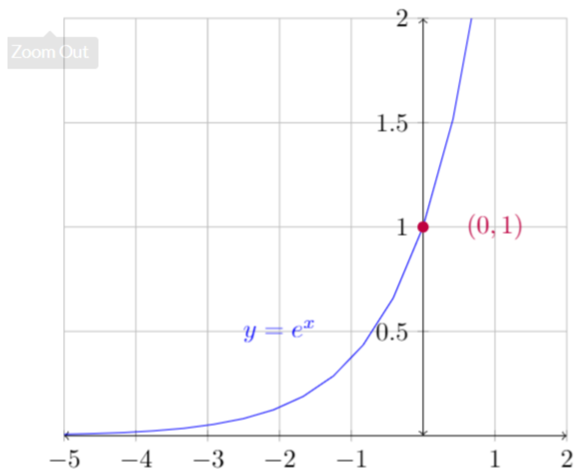
<u>Exercise 2:</u> Find the horizontal asymptote (HA) and the vertical asymptote (VA) for each of the following functions:

Hint: Instead of finding the HA and VA algebraically, find them using the graphs above. You may need to do a vertical/horizontal shift.

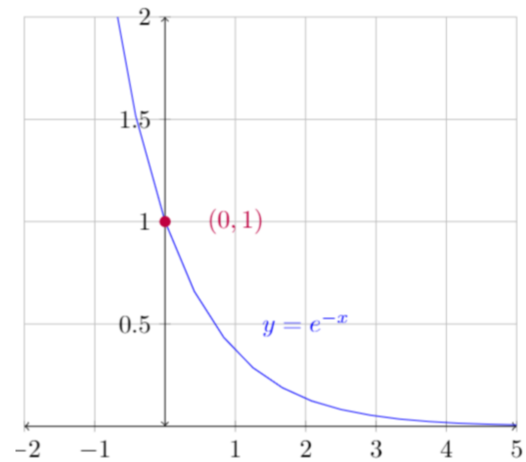
a) $y = \frac{1}{2x}$	**HA:**	**VA:**
b) $y = \frac{3}{x}$	**HA:**	**VA:**
c) $y = \frac{1}{x+5}$	**HA:**	**VA:**
d) $y = \frac{1}{x} - 4$	**HA:**	**VA:**
e) $y = \frac{2}{x^2}$	**HA:**	**VA:**
f) $y = \frac{1}{3x^2}$	**HA:**	**VA:**
g) $y = \frac{1}{(x+1)^2}$	**HA:**	**VA:**
h) $y = \frac{1}{x^2} + 10$	**HA:**	**VA:**
i) $y = e^x$	**HA:**	**VA:**
j) $y = e^{-x}$	**HA:**	**VA:**
k) $y = 3e^{-x}$	**HA:**	**VA:**
l) $y = 3e^{-x} + 3$	**HA:**	**VA:**
m) $y = 3e^{-x} + 1$	**HA:**	**VA:**

Y-INTERCEPTS

In the last section, we looked at these images. From them, we can see the following intercepts:



For $y = e^x$, y-intercept is $(0, 1)$



For $y = e^{-x}$, y-intercept is $(0, 1)$

Exercise 3: Find the y-intercept(s) for each of the following functions:

Hint: Instead of finding the y-intercept algebraically, find them using the graphs above. You may need to do a vertical/horizontal shift.

a) $y = e^x$ **Y-INTERCEPT:**

b) $y = e^{-x}$ **Y-INTERCEPT:**

c) $y = 3e^{-x}$ **Y-INTERCEPT:**

d) $y = 3e^{-x} + 3$ **Y-INTERCEPT:**

e) $y = 3e^{-x} + 1$ **Y-INTERCEPT:**

USEFUL DEFINITIONS FOR HW 18

1. Point at the origin ⇒ (0,0)

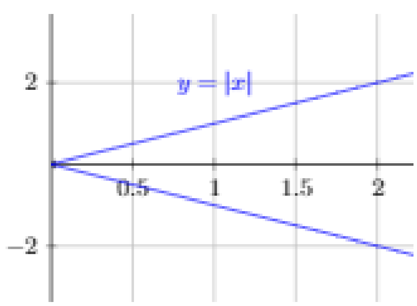
2. Lines ⇒ $y = mx + b$ where m is the slope and b is the y-intercept

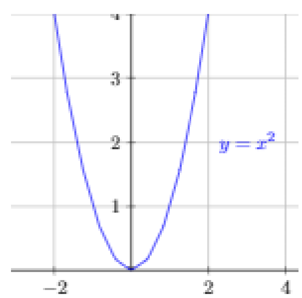
3. Parabolas ⇒ $y = a(x - h)^2 + k$ where (h, k) is the vertex of the parabola

4. Exponential Functions
   a. Increasing ⇒ example $y = e^x$
   b. Decreasing ⇒ example $y = e^{-x}$

5. Logarithmic Functions
   a. Increasing ⇒ example $y = ln\, x$
   b. Decreasing ⇒ example $y = -ln\, x$

6. Rational Functions are functions of the form: $y = \frac{p(x)}{q(x)}$
   a. x-axis symmetry ⇒ $f(x) = -f(x)$
   b. y-axis symmetry ⇒ $f(x) = f(-x)$





7. Circles ⇒ $(x - h)^2 + (y - k)^2 = r^2$ where r is radius and (h, k) is the center

8. Ellipses ⇒ $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where (h, k) is the center

9. Hyperbolas ⇒ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ where (h, k) is the center

To find the foci for 8 and 9, we use the equation $c^2 = a^2 + b^2$, and solve for c.

CIRCLES

Recall the formulas for Circles:

Equation of a Circle: $(\boldsymbol{x} - \boldsymbol{h})^2 + (\boldsymbol{y} - \boldsymbol{k})^2 = \boldsymbol{r}^2$

where $\boldsymbol{r}$ is the radius and $(\boldsymbol{h}, \boldsymbol{k})$ is the center of the circle

Area of the Circle: $A = \pi r^2$

Exercise 4: Find the area of the following circles:

a) $x^2 + y^2 = 1$ **Area = ____________**

b) $x^2 + y^2 = 3$ **Area = ____________**

c) A circle with center $(0,1)$ and radius 625 **Area = ____________**