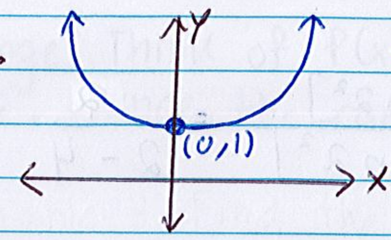
Lesson 18: Intro to Functions of Several Variables

Single Variable Functions

- Can be written as $\boxed{y=f(x)}$
  i.e. y is a function of x
- Takes as an input a # and produces as an output a #
- Graph: a curve in the xy-plane
  i.e. 2-D

ex. 

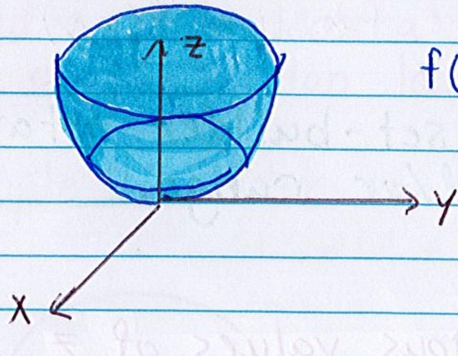
$f(x)=x^2+1$

Multivariable Functions

Definition: A multivariable function has 2 or more variables as inputs.

In this class, we are limitting ourselves to just 2 variables. So

- Can be written as $\boxed{z=f(x,y)}$
  i.e. z is a function of x and y
- Takes as an input a pair of # or a point and produces as an output a #
- Graph: A Surface in xyz-space
  i.e. 3-D

ex. 

$f(x,y)=x^2+y^2$

Recall the idea of cross sections and how stacking multiple 2-D objects became one 3-D object. Well this is a bunch of circles with increasing radii stacked on top of another.

Example 1: Compute the indicated functional value

ⓐ $f(x,y) = \dfrac{3x+2y}{2x+3y}$; $f(-4,6)$

$$f(-4,6) = \frac{3(-4)+2(6)}{2(-4)+3(6)} = \frac{-12+12}{-8+18} = \frac{0}{10} = 0$$

ⓑ $f(x,y) = \dfrac{e^{xy}}{2-e^{xy}}$; $f(\ln 2, 2)$ and $f(1, \ln 1)$

$$f(\ln 2, 2) = \frac{e^{2\ln 2}}{2-e^{2\ln 2}} = \frac{\exp[\ln 2^2]}{2-\exp[\ln 2^2]} = \frac{4}{2-4} = \frac{4}{-2} = -2$$

$$f(1, \ln 1) = \frac{e^{\ln 1}}{2-e^{\ln 1}} = \frac{1}{2-1} = \frac{1}{1} = 1$$

Domain and Range for Multivariable Functions

Just as for functions of one variables, we can find the domain and range of functions of two variable in a similar fashion. If you need a review of domain and range of one variable functions, look at the Algebra Review pdf for Lesson 18.

The main difference for two variable functions is

Domain: All points (x,y) in the xy-plane for which $f(x,y)$ is defined

Range: All values that the function $f(x,y)$ produces

Notation: We will be using set-builder notation for denoting the domain and/or range.

i.e. $\{x \mid x \geq 4\}$

Example 2: Describe the domain and range of the function

$$f(x,y) = \sqrt{x^2 - y}$$

Domain: Recall domain of $\sqrt{?}$ is $? \geq 0$. So

$$x^2 - y \geq 0$$
$$x^2 \geq y$$

Hence the domain is $\{(x,y) \mid x^2 \geq y\}$. Note this domain consists parabolas.

Range: Think of $f(x,y) = z$. Recall that the range of $\sqrt{\ }$ is ≥ 0. Hence the range is $\{z \mid z \geq 0\}$

Example 3: Find the domain of $f(x,y) = \dfrac{\sqrt{x-18}}{\ln(y-9)}$

To find the domain, we need to play with two portions of the function

- $\sqrt{x-18} \Rightarrow x - 18 \geq 0$, $x \geq 18$
- $\ln(y-9) \Rightarrow y - 9 > 0$, $y > 9$

Hence the domain is $\{(x,y) \mid x \geq 18, y > 9\}$.

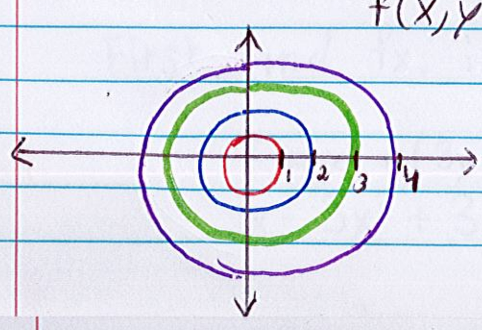
Level Curves

Definition: The level curves of a function of two variables are the curves with equations $f(x,y) = k$ where k is a constant (in the range of f).

The idea is to look at various values of z and see what each cross-section looks like.

Example 4: Sketch the indicated level curves

$$f(x,y) = x^2 + y^2, \quad C = 1, 4, 9, 16$$



$x^2 + y^2 = 1 = 1^2$

$x^2 + y^2 = 4 = 2^2$

$x^2 + y^2 = 9 = 3^2$

$x^2 + y^2 = 16 = 4^2$

Example 5: What do the level curves for $f(x,y) = 11\sqrt{y+6x^2}$ look like?

Let $f(x,y) = K$. So $K = 11\sqrt{y+6x^2}$. Now solve for y.

$$\frac{K}{11} = \sqrt{y+6x^2}$$

$$\left(\frac{K}{11}\right)^2 = y + 6x^2$$

$$\underbrace{\frac{K^2}{121}}_{\text{constant}} - 6x^2 = y$$

So the level curves are in the shapes of parabolas.