Example 2: Compute

@ $\int_2^4 x\sin(x^2)\,dx$ $\underset{du=2x\,dx}{\overset{u=x^2}{\longrightarrow}}$ $\int \sin(u)\frac{du}{2}$

$\frac{du}{2} = x\,dx$

Issue: What do I do w/ the limit values? i.e. $\int_2^4$

$$\int_2^4 x\sin(x^2)\,dx = \int_{x=2}^{x=4} x\sin(x^2)\,dx$$

Method 1: Is changing $x=2$ and $x=4$ using $u=x^2$

$x=2 \to u=4$ $\quad$ $x=4 \to u=16$

$$\int_{x=2}^{x=4} x\sin(x^2)\,dx \underset{\frac{du}{2}=x\,dx}{\overset{u=x^2}{\longrightarrow}} \int_{u=4}^{u=16} \sin(u)\frac{du}{2}$$

$$= -\frac{\cos(u)}{2}\Big]_4^{16} = \frac{-1}{2}\left(\cos(16)-\cos(4)\right)$$

Method 2: To treat the integral after the u-sub as indefinite and when you plug u back evaluate w/ original bounds.

$$\int_2^4 x\sin(x^2)\,dx \underset{du/2=x\,dx}{\overset{u=x^2}{\longrightarrow}} \int \frac{\sin(u)}{2}\,du$$

$$= -\frac{1}{2}\cos(u) = -\frac{1}{2}\cos(x^2)\Big]_2^4$$

$$= -\frac{1}{2}\left(\cos(16)-\cos(4)\right)$$