

# MA 16020 LESSON 21: DIFFERENTIALS OF MULTIVARIABLES FUNCTIONS (PROBLEM SET)

**Example 1:** Use increments to estimate the change in  $z$  at  $(x, y) = (0, 0)$  if

$$\frac{dz}{dx} = -8x + 6 \quad \text{and} \quad \frac{dz}{dy} = 6y + 4$$

The change in  $x$  is 0.3 and the change in  $y$  is 0.2.

**Example 2:** Find  $z = f(x, y)$  and use the total differential to approximate the quantity.

$$\sqrt{(5.3)^2 + (7.45)^2} - \sqrt{5^2 + 7^2}$$

**Example 3:** The specific gravity of an object with density greater than that of water can be determined by using the formula

$$S = \frac{A}{A - W}$$

where  $A$  and  $W$  are the weights of the object in air and in water, respectively. If the measurements of an object are  $A = 2.2$  lb and  $W = 2$  lb with maximum errors of 0.02 lb and 0.02 lb, respectively,

a) Find the approximate maximum error in calculating  $S$ .

b) Find the approximate relative percentage error in calculating  $S$ .

**Example 4:** Hot chocolate sales (in gallons) are predicted by

$$f(x, y) = 1.7x^{1/2}y^{-1}$$

where  $y$  is the temperature (in  $^{\circ}C$ ) and  $x$  is the amount of snow on the ground (in inches). If the temperature rises from  $1.6^{\circ}$  to  $1.7^{\circ}$  and the amount of snow falls from 2 inches to 1.2 inches, use differentials to estimate the change in hot chocolate sales.

a) The change in hot chocolate sales due to the change in the amount of snow is:

b) The change in hot chocolate sales due to the change in the temperature is:

c) The total change in hot chocolate sales is:

**Example 5:** A soft drink can is a cylinder  $h$  cm tall with radius  $r$  cm. It's volume is given by the formula

$$V(r, h) = \pi r^2 h$$

A particular can is 14 cm tall and has a radius of 4 cm. If the height is increased by 1.2 cm, use calculus to estimate the change in the radius needed so that the volume stays the same.