

MA 16020 LESSON 36: EIGENVALUES + EIGENVECTORS: 3x3 CASE (ALGEBRA REVIEW)

Unlike polynomials of degree 2 (i.e. quadratic polynomials), we don't have a formula for polynomials of degree 3 or more. Instead, we try to factor the polynomials. However, sometimes that isn't as easy as said.

When factoring isn't obvious, we can find the zeroes of these polynomials, we have 2 options:

- (1) Synthetic Division, or (2) Long Division

Since most of the homework problems can be done via factoring Synthetic Division, we WON'T be reviewing Long Division.

Example 1: Find all the solutions of the equation: $x^3 - 6x^2 + 11x - 6 = 0$

Method: Synthetic Division

To use Synthetic Division, we find need to find a zero of the polynomial. So choose a number and check that the equation is indeed zero at the x.

Let $x=1$.

$$1^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$$

So $x=1$ is a zero of the polynomial.

Example 1: Find all the solutions of the equation:

$$x^3 - 6x^2 + 11x - 6 = 0$$

Method: Synthetic Division

Now that we have one zero of the polynomial. Write that # at that left-most point of the "L" shape below.

$$x = 1$$



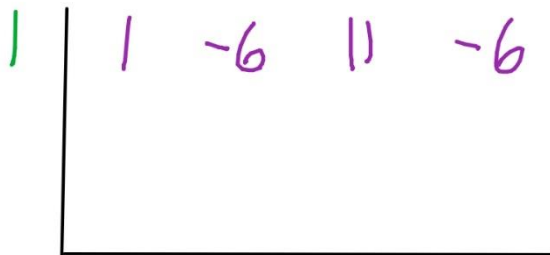
Example 1: Find all the solutions of the equation:

$$x^3 - 6x^2 + 11x - 6 = 0$$

Method: Synthetic Division

Next, list the coefficients of the polynomials adjacent to the zero we found.

$$x = 1$$



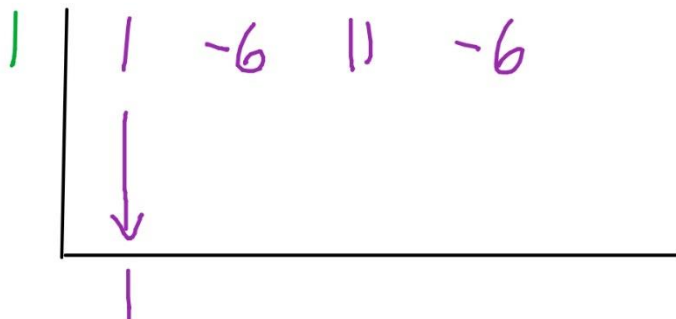
Example 1: Find all the solutions of the equation:

$$x^3 - 6x^2 + 11x - 6 = 0$$

Method: Synthetic Division

Drag down the first # inside the "L".

$$x = 1$$



Example 1: Find all the solutions of the equation:

$$x^3 - 6x^2 + 11x - 6 = 0$$

Method: Synthetic Division

Multiply the zero, $x=1$, we found earlier by the value just written on the bottom row. Place that number inside the "L" next to the downward arrow.

$$x = 1$$

1	1	-6	11	-6
	↓			
				1

Example 1: Find all the solutions of the equation:

$$x^3 - 6x^2 + 11x - 6 = 0$$

Method: Synthetic Division

Add the column created.

$$x = 1$$

1	1	-6	11	-6
	↓	↓		
				-5

Example 1: Find all the solutions of the equation: $x^3 - 6x^2 + 11x - 6 = 0$

Method: Synthetic Division

REPEAT: Multiply the zero, $x=1$, we found earlier by the value just written on the bottom row. Place that number inside the "L" next to the downward arrow.

$$x=1$$

1	1	-6	11	-6
↓		1		
1	-5			

(Note: Red arrows in the original image show the calculation of -5 from 11 and -6, and the multiplication of 1 by -5 to get -5.)

Example 1: Find all the solutions of the equation: $x^3 - 6x^2 + 11x - 6 = 0$

Method: Synthetic Division

REPEAT: Add the column created.

$$x=1$$

1	1	-6	11	-6
↓		1	-5	
1	-5	6		

(Note: Red arrows in the original image show the addition of 1 and -5 to get 6.)

Example 1: Find all the solutions of the equation: $x^3 - 6x^2 + 11x - 6 = 0$

Method: Synthetic Division

REPEAT: Multiply the zero, $x=1$, we found earlier by the value just written on the bottom row. Place that number inside the "L" next to the downward arrow.

$$x=1$$

1	1	-6	11	-6
↓		1	-5	6
	1	-5	6	

Example 1: Find all the solutions of the equation: $x^3 - 6x^2 + 11x - 6 = 0$

Method: Synthetic Division

REPEAT: Add the column created.

$$x=1$$

1	1	-6	11	-6
↓		1	-5	6
	1	-5	6	0

Example 1: Find all the solutions of the equation: $x^3 - 6x^2 + 11x - 6 = 0$

Method: Synthetic Division

SO WHAT DID WE FIND OUT? So the values beneath the "L" are the coefficients of the second factor of the original polynomial.

What is nice is that we can check that we choose the right zero by seeing the last entry being 0.

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & \downarrow & & & \\ & 1 & -5 & 6 & 0 \end{array}$$

$x = 1$

$x^2 \quad x \quad x^0 \quad \text{remainder}$

Example 1: Find all the solutions of the equation: $x^3 - 6x^2 + 11x - 6 = 0$

Hence

$$x^3 - 6x^2 + 11x - 6 = (x-1)(x^2 - 5x + 6)$$

To finish factoring, we can resort to easier means.

$$= (x-1)(x-2)(x-3)$$

Hence the solutions of the equation are:

$$x = 1, 2, 3$$

Exercise 1: Find all the solutions of the following equation:

Note that not all these examples require Synthetic Division.

1. $x^3 - 5x^2 - x + 5 = 0$

2. $x^3 - 2x^2 - 16x + 32 = 0$

3. $x^3 + 4x^2 - 36x - 144 = 0$

4. $x^3 - 2x^2 - 5x + 6 = 0$

5. $4x^3 - 7x + 3 = 0$

6. $x^3 - 4x^2 + 2x - 8 = 0$

7. $x^3 + 13x^2 + 32x + 20 = 0$

8. $x^3 + 3x^2 + x + 3 = 0$