Lesson 5: Integration By Parts

From the Problem Set (posted online),

Example 1: After t weeks, contributions in response to a local fund-raising campaign were coming in at the rate of $2000+e^{-0.2t}$ dollars per week. How much money was raised in the first 5 weeks?

$$\int_0^5 2000+e^{-0.2t}\,dt \qquad \begin{matrix} u=2000+ & dv=e^{-0.2t}dt \\ du=2000\,dt & v=\frac{1}{-0.2}e^{-0.2t} \\ & v=-5e^{-0.2t} \end{matrix}$$

$$=2000+(-5e^{-0.2t})\Big]_0^5-\int_0^5 -5e^{-0.2t}(2000)\,dt$$

$$=-10000+e^{-0.2t}\Big]_0^5+10000\int_0^5 e^{-0.2t}\,dt$$

$$=-10000+e^{-0.2t}\Big]_0^5+\frac{10000}{-0.2}e^{-0.2t}\Big]_0^5$$

$$=-10000+e^{-0.2t}\Big]_0^5-50000\,e^{-0.2t}\Big]_0^5$$

$$=-10000(5)e^{-0.2(5)}+10000(0)e^{-0.2(0)}\to 0$$

$$-50000e^{-0.2(5)}+50000\underbrace{e^{-0.2(0)}}_{1}$$

$$=-10000\,e^{-1}+50000$$

$$\approx \$13212.06$$

Example 2: Find the area bounded by the curves $y=9x^3\ln x$ $y=0$ $x=1$ $x=7$

$$\int_1^7 9x^3\ln x\,dx \qquad \begin{matrix} u=\ln x & dv=9x^3dx \\ du=\frac{1}{x}dx & v=\frac{9}{4}x^4 \end{matrix}$$

$$= \frac{9}{4}x^4 \ln x \Big]_1^7 - \int_1^7 \frac{9}{4}x^4 \frac{dx}{x}$$

$$= \frac{9}{4}x^4 \ln x \Big]_1^7 - \int_1^7 \frac{9}{4}x^3 dx = \frac{9}{4}x^4 \ln x \Big]_1^7 - \frac{9}{4}\cdot\frac{x^4}{4} \Big]_1^7$$

$$= \frac{9}{4}(7)^4 \ln(7) - \frac{9}{4}(1)^4 \ln(1) - \frac{9}{16}(7)^4 + \frac{9}{16}(1)^4$$

$$= \frac{9}{4}(7)^4 \ln(7) - \frac{9}{16}(7)^4 + \frac{9}{16} \approx 9162.29$$

HW 5.8: When samples of iron ore are tested for potential mining sites, the probability (0 to 1) of finding a sample that has x percentage of iron in the sample is described by $\frac{49}{22}\cdot\frac{x}{\sqrt{1+5x}}$ where x is also between 0 and 1. Find the probability that a tested sample of iron ore is at least 79% iron.

At least 79% $\Rightarrow (0.79, 1)$

Note you can do this w/ a u-sub.

$$\int_{0.79}^{1} \frac{49}{22} x(1+5x)^{-1/2}\,dx \qquad \begin{matrix} u = 1+5x \iff (u-1)/5 = x \\ du = 5dx \iff du/5 = dx \end{matrix}$$

$$= \int \frac{49}{22}\cdot\frac{(u-1)}{5}u^{-1/2}\frac{du}{5} = \int \frac{49}{550}\left(u^{1/2} - u^{-1/2}\right)du$$

$$= \frac{49}{550}\left(\frac{2}{3}u^{3/2} - \frac{2}{1}u^{1/2}\right)$$

$$= \frac{49}{550}\left(\frac{2}{3}(1+5x)^{3/2} - 2(1+5x)^{1/2}\right)\Big]_{0.79}^{1}$$

$$= \frac{49}{550}\left(\frac{2}{3}(1+5)^{3/2} - 2(1+5)^{1/2}\right) - \frac{49}{550}\left(\frac{2}{3}(1+5(0.79))^{3/2} - 2(1+5(0.79))^{1/2}\right)$$

$$\approx 0.1788$$