

# Lesson 9: First-Order Linear Differential Equations

Warm-Up: Compute the following derivatives:

(a)  $\frac{d}{dt} \left( \int P(t) dt \right)$

By the Fundamental Theorem of Calculus (FTC),

$$\frac{d}{dt} \left( \int P(t) dt \right) = P(t)$$

(b)  $\frac{d}{dt} \left( e^{\int P(t) dt} \right)$

By the Chain Rule,

$$\frac{d}{dt} \left( e^{\int P(t) dt} \right) = e^{\int P(t) dt} \cdot \frac{d}{dt} \left( \int P(t) dt \right)$$

by (a)  $e^{\int P(t) dt} \cdot P(t)$

(c)  $\frac{d}{dt} \left( y(t) e^{\int P(t) dt} \right)$

By the Product Rule,

$$\frac{d}{dt} \left( y(t) e^{\int P(t) dt} \right) = y'(t) e^{\int P(t) dt} + y(t) \cdot \frac{d}{dt} \left( e^{\int P(t) dt} \right)$$

by (b)  $y'(t) e^{\int P(t) dt} + y(t) P(t) e^{\int P(t) dt}$