MATH 16020: Applied Calculus 2 Quiz 10 (Solutions) Mon., Mar. 7, 2022

> Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:______________________________

1. **[4 pts]** Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

Solution: To find the domain of $f(x,y)$, we need to solve the following inequalities:

- **[1 pt]** $\sqrt{x+y-1} \Rightarrow x+y-1 \geq 0$
  $y \geq 1-x$
- **[1 pt]** $\ln(y-11) \Rightarrow y-11 > 0$
  $y > 11$
- **[1 pt]** $\ln(y-11)-9 \neq 0$
  $\ln(y-11) \neq 9$
  $y-11 \neq e^9$
  $y \neq e^9+11$

Hence when we put them all together, the domain is

$$\{(x,y) \mid y \geq 1-x, \;\; y > 11, \;\; y \neq e^9+11\} \quad \textbf{[1 pt]}$$

2. Describe the sketch of the level curves of the function for the function

$$f(x,y) = \ln(y-e^{5x})$$

(a) **[2 pts]** Find the general equation for the level curves.

Solution: Let $k = f(x,y)$. Now solve for y.

$$k = \ln(y-e^{5x}) \quad \textbf{[1 pt]}$$
$$e^k = y - e^{5x}$$
$$y = e^{5x}+e^k \quad \textbf{[1 pt]}$$

(b) **[1 pt]** What functions $y = f(x)$, for you get for these values $z = 0, \ln(10)$?

Solution: Using (a),

- **[0.5 pt]** When $z = 0$,

$$y = e^{5x} + e^0 = e^{5x} + 1$$

- **[0.5 pt]** When $z = \ln(10)$,

$$y = e^{5x} + e^{\ln 10} = e^{5x} + 10$$

(c) **[1 pt]** What type of function describes the level curves?

(i) Increasing exponential functions
(ii) Rational Functions with x-axis symmetry
(iii) Natural logarithm functions
(iv) Decreasing exponential functions
(v) Rational Functions with y-axis symmetry

Solution: **[1 pt]** By (b), the type of function that describes the level curve is (i) Increasing exponential functions

(d) **[1 pt]** Determine the Horizontal Asymptotes (HAs) of the level curves found in (b).

Solution: Recall the HA of $y = e^{ax} + m$ is $y = m$. Using the functions from (b),

- **[0.5 pt]** The HA of $y = e^{5x} + 1$ is

$$y = 1$$

- **[0.5 pt]** The HA of $y = e^{5x} + 10$ is

$$y = 10$$

(e) **[1 pt]** Sketch the level curves found in (b).

Solution: