MATH 16020: Applied Calculus 2 Quiz 11 (Solutions) Wed., Mar. 23, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

1. **[5 pts]** Find a function $z = f(x, y)$ and then use the total differential to approximate the quantity to 3 decimals.

$$\cos\left(\frac{\pi}{4} + 0.02\right)\sin\left(\frac{\pi}{4} - 0.03\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$$

Solution: **[1 pt]** Let's make $f(x, y) = \cos(x)\sin(y)$.

[1 pt] So $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$, $\Delta x = 0.02$, and $\Delta y = -0.03$.

[1 pt] First, let's find the partials of V.

$$f_x(x, y) = -\sin(x)\sin(y) \qquad f_y(x, y) = \cos(x)\cos(y)$$

[1 pt] Next, let's plug $x = \frac{\pi}{4}$ and $y = \frac{\pi}{4}$ into the partials.

$$f_x\left(\frac{\pi}{4}, \frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) = -\frac{1}{2} \qquad f_y\left(\frac{\pi}{4}, \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

Using the formula,

$$\Delta f = f_x\left(\frac{\pi}{4}, \frac{\pi}{4}\right)\Delta x + f_y\left(\frac{\pi}{4}, \frac{\pi}{4}\right)\Delta y$$

plug in the values $f_x\left(\frac{\pi}{4}, \frac{\pi}{4}\right) = -\frac{1}{2}$, $f_y\left(\frac{\pi}{4}, \frac{\pi}{4}\right) - \frac{1}{2}$, $\Delta x = 0.02$, and $\Delta y = -0.03$.

$$\Delta f = -\frac{1}{2}(0.02) + \frac{1}{2}(-0.03) = -0.025 \qquad \textbf{[1 pt]}$$

2. **[5 pts]** A soft drink can is a cylinder h cm tall with radius r cm. It's volume is given by the formula

$$V(r, h) = \pi r^2 h$$

A particular can is 14 cm tall and has a radius of 4 cm. If the height is **DECREASED** by 1.2 cm, use calculus to estimate the change in the radius needed so that the volume stays the same. Round your answer to 4 decimal places.

Solution: This problem wants us to find Δr such that $\Delta V = 0$.

[1 pt] We are given $h = 14$, $r = 4$, and $\Delta h = -1.2$.

[1 pt] First, let's find the partials of V.

$$V_r(r,h) = 2\pi rh \qquad\qquad V_h(r,h) = \pi r^2$$

[1 pt] Next, let's plug $r = 4$ and $h = 14$ into the partials.

$$V_r(4,14) = 112\pi \qquad\qquad V_h(4,14) = 16\pi$$

Using the formula,

$$\Delta V = V_r(4,14)\Delta r + V_h(4,14)\Delta h$$

plug in the values $V_r(4,14) = 112\pi$, $V_h(4,14) = 16\pi$, and $\Delta h = -1.2$

$$\Delta V = 112\pi\Delta r + 16\pi(-1.2)$$

Now set this equation equal to 0 and solve for Δr.

$$0 = 112\pi\Delta r - 19.2\pi \qquad \textbf{[1 pt]}$$

$$\Delta r = \frac{19.2\pi}{112\pi} = 0.1714 \qquad \textbf{[1 pt]}$$