MATH 16020: APPLIED CALCULUS 2 QUIZ 12 (SOLUTIONS) MON., MAR. 28, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:____

1. **[6 pts]** Evaluate $\frac{dz}{dt}$ at $t = 1$ if

$$z = \exp[x^2 + 4xy + y^2 + 3y] \qquad x = \cos\left(\frac{\pi}{2}t\right) \qquad y = \ln t$$

Solution: **[2 pts]** First find the partials of z

$$\frac{dz}{dx} = (2x + 4y)\exp[x^2 + 4xy + y^2 + 3y] \qquad \frac{dz}{dy} = (4x + 2y + 3)\exp[x^2 + 4xy + y^2 + 3y]$$

[1 pt] Next find the derivatives of x and y

$$\frac{dx}{dt} = -\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right) \qquad \frac{dy}{dt} = \frac{1}{t}$$

[2 pts] Before plugging all the equations into the multivariate chain rule, let's plug in $t = 1$. So,

$$x(1) = \cos\left(\frac{\pi}{2}\right) = 0 \qquad y(1) = \ln(1) = 0$$

$$\left.\frac{dx}{dt}\right|_{(0,0)} = -\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} \qquad \left.\frac{dy}{dt}\right|_{(0,0)} = \frac{1}{1} = 1$$

$$\left.\frac{dz}{dx}\right|_{(0,0)} = (2(0) + 4(0))\exp(0) = 0 \qquad \left.\frac{dz}{dy}\right|_{(0,0)} = (4(0) + 2(0) + 3)\exp(0) = 3$$

By the multivariate chain rule,

$$\left.\frac{dz}{dt}\right|_{t=1} = \left.\frac{dz}{dx}\right|_{(x,y)=(0,0)} \left.\frac{dx}{dt}\right|_{t=1} + \left.\frac{dz}{dy}\right|_{(x,y)=(0,0)} \left.\frac{dy}{dt}\right|_{t=1} = 3 \;\; \textbf{[1 pt]}$$

2. **[5 pts]** The surface area of a cylinder is given by

$$A(h, r) = 2\pi r^2 + 2\pi rh$$

where h is the height of the cylinder and r is the radius. Suppose

- the height of the cylinder is decreasing at a rate of 4 inches per minute
- the radius of the cylinder is increasing at a rate of 2 inches per minute.

What is the rate of change of the surface area when the height is 10 inches and the radius is 15 inches?

Solution: **[1 pt]** We are given $h = 10, \quad r = 15, \quad \Delta h = -4 \quad \text{and} \quad \Delta r = 2.$

[1 pt] First, let's find the partials of A.

$$A_r(r, h) = 4\pi r + 2\pi h \qquad\qquad A_h(r, h) = 2\pi r$$

[1 pt] Next, let's plug $h = 10$ and $r = 15$ into the partials.

$$A_r(10, 15) = 60\pi + 20\pi = 80\pi \qquad\qquad A_h(10, 15) = 30\pi$$

Using the formula,

$$\begin{aligned} \Delta A &= A_r(10, 15)\Delta r + A_h(10, 15)\Delta h \\ &= 80\pi \cdot (2) + 30\pi \cdot (-4) = 40\pi \qquad \textbf{[1 pt]} \end{aligned}$$