Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name:____

1. [**5 pts**] Find the discriminant of

$$f(x,y) = e^x \sin(y)$$

Simplify your answer. Note: $\sin^2(y) + \cos^2(y) = 1$.

Solution: [**1 pt**] Recall that the discriminant is

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

[**3 pts**]First find the partials:

$$f_x = e^x \sin(y) \qquad\qquad f_y = e^x \cos(y)$$

$$f_{xx} = e^x \sin(y) \qquad f_{xy} = e^x \cos(y) \qquad f_{yy} = -e^x \sin(y)$$

Now plug the correct partials into the discriminant formula above:

$$\begin{aligned} D &= e^x \sin(y) \cdot (-e^x \sin(y)) - (e^x \cos(y))^2 \\ &= -(e^x \sin(y))^2 - (e^x \cos(y))^2 \\ &= -e^{2x} \sin^2(y) - e^{2x} \cos^2(y) \\ &= -e^{2x}(\sin^2(y) + \cos^2(y)) \\ &= -e^{2x} \qquad\qquad \textbf{[1 pt]} \end{aligned}$$

2. **[5 pts]** Given the information in the table below, find and classify any critical points for the function $g(x, y)$.

(x_0, y_0)	$g(x_0, y_0)$	$g_x(x_0, y_0)$	$g_y(x_0, y_0)$	$g_{xx}(x_0, y_0)$	$g_{xy}(x_0, y_0)$	$g_{yy}(x_0, y_0)$
$(0, 1)$	0	3	0	0	-2	4
$(4, 3)$	-3	0	0	-1	2	-6
$(2, 7)$	15	0	0	4	5	8
$(5, 6)$	4	0	0	3	5	2
$(-2, 8)$	2	0	0	2	2	2

Solution: First check for each point that both g_x and g_y are 0.

- Hence $(0, 1)$ is not a critical point. **[1 pt]**

Next, let's compute the discriminant of each point.

- $\underline{(4, 3)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = -1 \cdot (-6) - (2)^2 = -2$
- $\underline{(2, 7)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 4 \cdot 8 - (5)^2 = 7$
- $\underline{(5, 6)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 3 \cdot 2 - (5)^2 = -19$
- $\underline{(-2, 8)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 2 \cdot 2 - (2)^2 = 0$

When $D > 0$, we have a relative extrema. Hence $(4, 3)$ and $(2, 7)$ are relative extrema. To determine whether they are maxs or mins, we need to check the sign of g_{xx}.

- $\underline{(4, 3)}$: $g_{xx} = -1 < 0$. Hence $(4, 3)$ is a relative max. **[1 pt]**
- $\underline{(2, 7)}$: $g_{xx} = 4 > 0$. Hence $(2, 7)$ is a relative min. **[1 pt]**

When $D < 0$, we have a saddle point. Hence $(5, 6)$ is a saddle point. **[1 pt]**
When $D = 0$, the test is inconclusive. Hence at $(-2, 8)$ the test is inconclusive. **[1 pt]**