MATH 16020: Applied Calculus 2 Quiz 14 (Solutions) Mon., April 4, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:____

1. **[4 pts]** Find the point(s) (x, y) where the function $f(x, y) = 3x^2 + 4xy + 6x - 15$ attains maximal value, subject to the constraint $x + y = 10$.

Solution: Here $g(x, y) = x + y = 10$. Using LaGrange Multipliers,

$$f_x = 6x + 4y + 6 \qquad \lambda g_x = \lambda(1) = \lambda$$
$$f_y = 4x \qquad \lambda g_y = \lambda(1) = \lambda$$

So our system is

$$\begin{cases} 6x + 4y + 6 = \lambda & (1) \\ 4x = \lambda & (2) \\ x + y = 10 & (3) \end{cases} \qquad \textbf{[1 pt]}$$

[1 pt] Set (1) and (2) equal to each other.

$$6x + 4y + 6 = \lambda = 4x$$
$$2x + 4y + 6 = 0$$
$$x + 2y + 3 = 0$$
$$x = -2y - 3$$

[1 pt] Plug $x = -2y - 3$ into (3).

$$x + y = 10$$
$$-2y - 3 + y = 10$$
$$-y - 3 = 10$$
$$y + 3 = -10y = -13$$

[1 pt] Plug $y = -13$ into $x = -2y - 3$.

$$x = -2y - 3$$
$$x = -2(-13) - 3$$
$$x = 23$$

2. **[6 pts]** Find the minimum of the function using **LaGrange Multipliers** of the function $f(x, y) = 2x^2 + 4y^2$ subject to the constraint $x^2 + y^2 = 1$.

Solution: Here $g(x, y) = x^2 + y^2 = 1$. Using LaGrange Multipliers,

$$f_x = 4x \qquad \lambda g_x = \lambda(2x) = \lambda$$
$$f_y = 8y \qquad \lambda g_y = \lambda(2y) = \lambda$$

So our system is

$$\begin{cases} 4x = 2x\lambda & (1) \\ 8y = 2y\lambda & (2) \\ x^2 + y^2 = 1 & (3) \end{cases} \qquad \textbf{[1 pt]}$$

[1 pt] Solve (1).

$$4x = 2x\lambda$$
$$4x - 2x\lambda = 0$$
$$2x(2 - \lambda) = 0$$

So $x = 0$ or $\lambda = 2$.

[1 pt] Plug $x = 0$ into (3).

$$x^2 + y^2 = 1$$
$$0^2 + y^2 = 1$$
$$y = \pm 1$$

[1 pt] Plug $\lambda = 2$ into (2).

$$8y - 2y\lambda = 0$$
$$8y - 4y = 0$$
$$4y = 0$$
$$y = 0$$

[1 pt] Plug $y = 0$ into (3).

$$x^2 + y^2 = 1$$
$$x^2 + 0 = 1$$
$$x = \pm 1$$

Hence all the points that need to be tested are $(1, 0), (-1, 0), (0, 1), (0, -1)$. So

- $f(1, 0) = 2(1)^2 + 4(0)^2 = 2$
- $f(-1, 0) = 2(-1)^2 + 4(0)^2 = 2$
- $f(0, 1) = 2(0)^2 + 4(1)^2 = 4$
- $f(0, -1) = 2(0)^2 + 4(-1)^2 = 4$

[1 pt] Hence the minimum value is 2.