

MATH 16020: APPLIED CALCULUS 2 QUIZ 15 (SOLUTIONS) MON., APRIL 18, 2022

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name: _____

1. [5 pts] Find all solutions **via Gauss Elimination** to the system of linear equations.

$$\begin{cases} 3x + 2y = 4 \\ 2x + 5y = 1 \end{cases}$$

Solution: Rewrite the system into an augmented matrix, and get it in row echelon form. **Gauss Elimination.**

$$\begin{aligned} \left[\begin{array}{cc|c} 3 & 2 & 4 \\ 2 & 5 & 1 \end{array} \right] & \xrightarrow{R_1 - R_2 \rightarrow R_1} \left[\begin{array}{cc|c} 1 & -3 & 3 \\ 2 & 5 & 1 \end{array} \right] \\ & \xrightarrow{2R_1 - R_2 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & -11 & 5 \end{array} \right] \\ & \xrightarrow{(-1/11)R_2 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & 1 & -5/11 \end{array} \right] \end{aligned}$$

Hence we have the following system:

$$\begin{cases} x - 3y = 3 & (1) \\ y = -5/11 & (2) \end{cases}$$

Plug $y = -5/11$ into (1)

$$\begin{aligned} x - 3\left(\frac{-5}{11}\right) &= 3 \\ x + \left(\frac{15}{11}\right) &= 3 \\ x &= 3 - \left(\frac{15}{11}\right) = \frac{18}{11} \end{aligned}$$

Hence the solution to this system is $(18/11, -5/11)$.

How I graded?

- 1 pt for augmented matrix
- 2 pts for get the matrix in row echelon form
- 1 pt for the new system
- 1 pt for the solution

2. [5 pts] Reduce the augmented matrix into a **REDUCED** row echelon form:

$$\left[\begin{array}{ccc|c} 4 & -3 & -2 & 7 \\ 3 & 0 & 2 & 8 \\ 2 & 3 & -1 & 2 \end{array} \right]$$

Solution: Your process will probably differ from the solution provided below. If so, as long as you used valid row operations, points will not be taken off.

$$\begin{aligned} \left[\begin{array}{ccc|c} 4 & -3 & -2 & 7 \\ 3 & 0 & 2 & 8 \\ 2 & 3 & -1 & 2 \end{array} \right] &\xrightarrow{R_1 - R_2 \rightarrow R_1} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -1 \\ 3 & 0 & 2 & 8 \\ 2 & 3 & -1 & 2 \end{array} \right] \\ &\xrightarrow{R_2 - R_3 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -1 \\ 1 & -3 & 3 & 6 \\ 2 & 3 & -1 & 2 \end{array} \right] \\ &\xrightarrow{R_1 - R_2 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -1 \\ 0 & 0 & -7 & -7 \\ 2 & 3 & -1 & 2 \end{array} \right] \\ &\xrightarrow{\substack{(-1/7)R_2 \rightarrow R_2 \\ R_2 \leftrightarrow R_3}} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -1 \\ 2 & 3 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right] \\ &\xrightarrow{2R_1 - R_2 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -1 \\ 0 & -9 & -7 & -4 \\ 0 & 0 & 1 & 1 \end{array} \right] \\ &\xrightarrow{R_2 + 7R_3 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & -3 & -4 & -1 \\ 0 & -9 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array} \right] \\ &\xrightarrow{R_1 + 4R_3 \rightarrow R_1} \left[\begin{array}{ccc|c} 1 & -3 & 0 & 3 \\ 0 & -9 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{array} \right] \\ &\xrightarrow{(-1/9)R_2 \rightarrow R_2} \left[\begin{array}{ccc|c} 1 & -3 & 0 & 3 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 1 \end{array} \right] \\ &\xrightarrow{R_1 + 3R_2 \rightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 1 \end{array} \right] \end{aligned}$$

How I graded?

- 1 pt for trying
- 1 pt for getting the matrix in reduced echelon form
- 1 pt for each right answer