MATH 16020: Applied Calculus 2 Quiz 16 (Solutions) Fri., April 22, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:______________________________________

1. Find the inverse matrix of following matrices:

(a) **[4 pts]** $A = \begin{bmatrix} 2 & 4 \\ 1 & 6 \end{bmatrix}$

Solution: **[1 pt]** Recall the formula for finding the inverse matrix of a 2×2 matrix:

$$\textit{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \textit{ then } A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Hence

$$A^{-1} = \frac{1}{2(6) - 4(1)} \begin{bmatrix} 6 & -4 \\ -1 & 2 \end{bmatrix} \qquad \textbf{[1 pt]}$$

$$= \frac{1}{8} \begin{bmatrix} 6 & -4 \\ -1 & 2 \end{bmatrix} \qquad \textbf{[1 pt]}$$

$$= \begin{bmatrix} 6/8 & -4/8 \\ -1/8 & 2/8 \end{bmatrix} \qquad \textbf{[1 pt]}$$

(b) **[6 pts]** $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$

Solution: Recall that there is no formula to find the inverse of a 3×3 matrix. Hence we need to calculate it via an augment matrix:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 5 & 6 & 0 & 0 & 0 & 1 \end{array}\right] \qquad \textbf{[1 pt]}$$

[4 pts] Your process will probably differ from the solution provided below. If so, as long as you used valid row operations, points will not be taken off.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 5 & 6 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{5R_1 - R_3 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 4 & 15 & 5 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{4R_2 - R_3 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - 4R_3 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 20 & -15 & -4 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 - 2R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & -39 & 30 & 8 \\ 0 & 1 & 0 & 20 & -15 & -4 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 - 3R_3 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -24 & 18 & 5 \\ 0 & 1 & 0 & 20 & -15 & -4 \\ 0 & 0 & 1 & -5 & 4 & 1 \end{array}\right]$$

Hence

$$B^{-1} = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix} \qquad \textbf{[1 pt]}$$