MATH 16020: Applied Calculus 2 Quiz 5 (Solutions) Fri., Feb. 11, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

1. **[4 pts]** Solve for y as a function of t when $y' = 20 \ln t$.

Solution: Rewrite:

$$y' = 20 \ln t$$
$$\frac{dy}{dt} = 20 \ln t$$
$$dy = 20 \ln t \, dt \qquad \textbf{[1 pt]}$$

Integrate:

$$\int dy = \int 20 \ln t \, dt$$
$$y = \int 20 \ln t \, dt$$

Use Integration by Parts on the integral on the RHS. **[1 pt]**

$$u = 20 \ln t \qquad dv = dt$$

$$du = \frac{20}{t} \, dt \qquad v = t$$

So:

$$y = 20t \ln t - \int \frac{20}{t} \cdot t \, dt \qquad \textbf{[1 pt]}$$
$$y = 20t \ln t - \int 20 \, dt$$
$$y = 20t \ln t - 20t + C \qquad \textbf{[1 pt]}$$

2. **[6 pts]** Consider the following IVP: $\frac{dy}{dx} = 11x^2 e^{-x^3}$ where $y = 10$ when $x = 2$. Find the value of the integration constant, C.

Solution: Rewrite:

$$\frac{dy}{dx} = 11x^2 e^{-x^3}$$
$$dy = 11x^2 e^{-x^3} \, dx \qquad \textbf{[1 pt]}$$

Integrate:

$$\int dy = \int 11x^2 e^{-x^3} \, dx$$
$$y = \int 11x^2 e^{-x^3} \, dx$$

Use u-substitution on the integral on the RHS. **[1 pt]**

$$\begin{array}{c} u = -x^3 \\ \hline\hline du = -3x^2\,dx \\ \dfrac{du}{-3x} = x^2\,dx \end{array}$$

So:

$$y = -\frac{11}{3}\int e^u\,du \qquad \textbf{[1 pt]}$$

$$y = -\frac{11}{3}e^{-x^3} + C \qquad \textbf{[1 pt]}$$

Now we need to find C. Using the fact $y = 10$ and $x = 2$,

$$10 = -\frac{11}{3}e^{-2^3} + C \qquad \textbf{[1 pt]}$$

$$C = 10 + \frac{11}{3}e^{-8} \qquad \textbf{[1 pt]}$$