MATH 16020: Applied Calculus 2 Quiz 7 (Solutions) Fri., Feb. 25, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

1. **[5 pts] (EXACT ANSWER ONLY!)** Find the volume of the solid generated by revolving the region enclosed by the curves

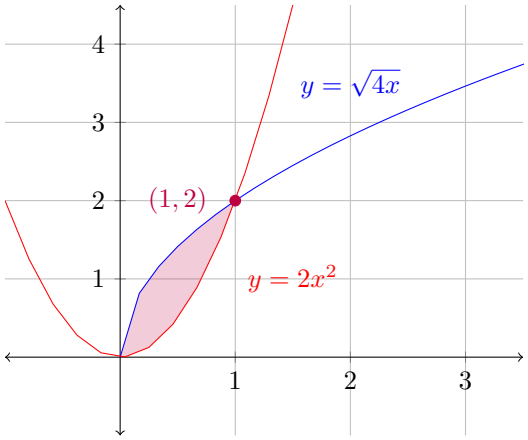
$$y = \sqrt{4x} \quad \text{and} \quad y = 2x^2 \quad \text{about the } y\text{-axis}$$

Solution: WASHER PROBLEM

Since we are doing rotation about the y-axis, we need to determine the Right and Left functions.

$$\text{Right} \quad y = 2x^2 \quad \longleftrightarrow \quad x = \sqrt{\frac{y}{2}}$$

$$\text{Left} \quad y = \sqrt{4x} \quad \longleftrightarrow \quad x = \frac{y^2}{4}$$



To find the bounds of the integral, set both equations equal.

$$\begin{aligned}
\sqrt{\frac{y}{2}} &= \frac{y^2}{4} \\
\frac{y}{2} &= \frac{y^4}{16} \\
y^4 &= 8y \\
y^4 - 8y &= 0 \\
y(y^3 - 8) &= 0 \\
y &= 0, \quad 2
\end{aligned}$$

From the graph and the previous calculations, we get the formula

$$\begin{aligned}
V &= \pi \int_0^2 \left[\left(\sqrt{\frac{y}{2}} \right)^2 - \left(\frac{y^2}{4} \right)^2 \right] dy \\
&= \pi \int_0^2 \left[\frac{y}{2} - \frac{y^4}{16} \right] dy \\
&= \pi \left[\frac{y^2}{4} - \frac{y^5}{16(5)} \right]_0^2 \\
&= \pi \left[\frac{2^2}{4} - \frac{2^5}{16(5)} \right] \\
&= \frac{3}{5}\pi
\end{aligned}$$

How I graded?

- **3 pts** for Set-Up
- **1 pt** for Integration
- **1 pt** for final answer

2. **[5 pts] (EXACT ANSWER ONLY!)** Find the volume of the solid generated by revolving the region enclosed by the curves

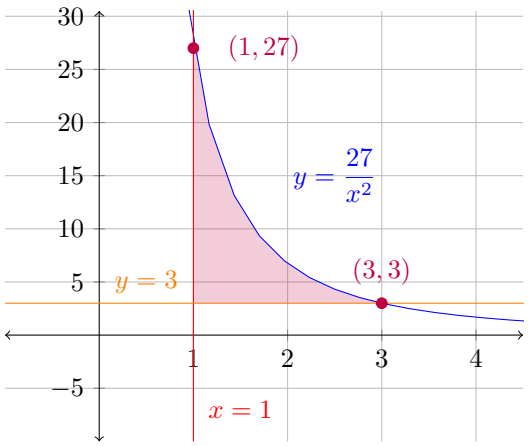
$$y = \frac{27}{x^2}\ , \quad y = 3 \quad \text{and} \quad x = 1 \quad \text{about the line } y = 3$$

Solution: DISK PROBLEM

Since we are doing rotation about the line $y = 3$ (parallel to x-axis) and the region "hugs" the line $y = 3$, we just need to find

$$f(x) - 3 = \frac{27}{x^2} - 3$$

Now to find the bounds of the integral. Note the smallest value of x is 1. Now to find the biggest value, plug $y = 3$ into $y = \frac{27}{x^2}$.



$$\begin{aligned} 3 &= \frac{27}{x^2} \\ 3x^2 &= 27 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

Note that we discard $x = -3$ because of the graph.

From the graph and above, we get the formula

$$\begin{aligned} V = \pi \int_1^3 \left(\frac{27}{x^2} - 3\right)^2 dx &= \pi \int_1^3 \left[\frac{729}{x^4} - \frac{162}{x^2} + 9\right] dx \\ &= \pi \int_1^3 \left[729x^{-4} - 162x^{-2} + 9\right] dx \\ &= \pi \left[\frac{729}{-3}x^{-3} - \frac{162}{-1}x^{-1} + 9x\right]_1^3 \\ &= \pi \left[-\frac{243}{x^3} + \frac{162}{x} + 9x\right]_1^3 \\ &= 144\pi \end{aligned}$$

How I graded?

- **3 pts** for Set-Up
- **1 pt** for Integration
- **1 pt** for final answer